Φ

Fee-fi-fo-fum: The Golden Ratio is One Giant of a Number

Kentucky Mathematical Association of Two-Year Colleges

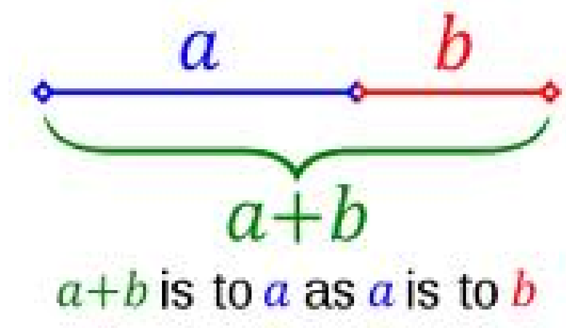
Jim Ham

Delta College

AMATYC

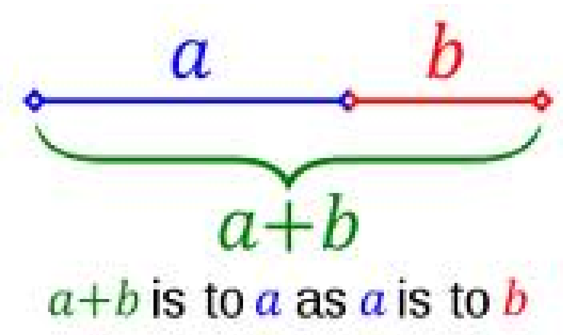
40th Annual KYMATYC Conference
Lake Cumberland State Park Resort
Friday, February 28, 2014

Φ



$$\frac{a+b}{a}=\frac{a}{b}$$

Φ

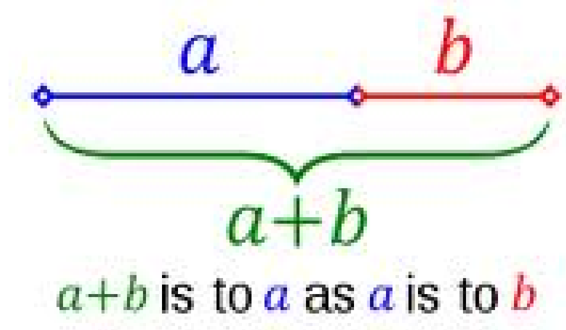


$$\frac{a+b}{a}=\frac{a}{b}$$

The Golden Ratio is also sometimes called

- **The golden section**
- **The golden mean**
- **The golden number**
- **The divine proportion**
- **The divine section**
- **The golden proportion**

Φ



$$\frac{a+b}{a}=\frac{a}{b}$$

$$If\ b=1\ldots$$

$$\frac{a+1}{a}=\frac{a}{1}$$

$$a^2=a+1$$

$$a^2-a-1=0$$

$$a=\frac{1\pm\sqrt{5}}{2}$$

Φ

$$\Phi = \frac{1+\sqrt{5}}{2}$$

1.61803398874989484820458683436563811772030917980576286213544⋱
86227052604628189024497072072041893911374847540880753868917⋱
52126633862223536931793180060766726354433389086595939582⋱
90563832266131992829026788067520876689250171169620703222104⋱
3216269548626296313614438149758701220340805887954454749246⋱
18569536486444924104432077134494704956584678850987433944221⋱
2544877066478091588460749988712400765217057517978834166256⋱
2494075890697040002812104276217711177780531531714101170466659914669798731761356006708748071013179523689427521948435305678300228785699782977834784587822891109762500302696156170025046433824377648610283831268330372429267526311653392473167111211588186385133162038400522216579128667529465490681131715993432359734949850904094762132229810172610705961164562990981629055520852479035240602017279974717534277759277862561943208275051312181562855122248093947123414517022373580577278616008688382952304592647878017889921990270776903895321968198615143780314997411069260886742962267575605231727775203536139362...

Φφ

$\Phi = 1.618033989...$

$\varphi = -0.618033989...$

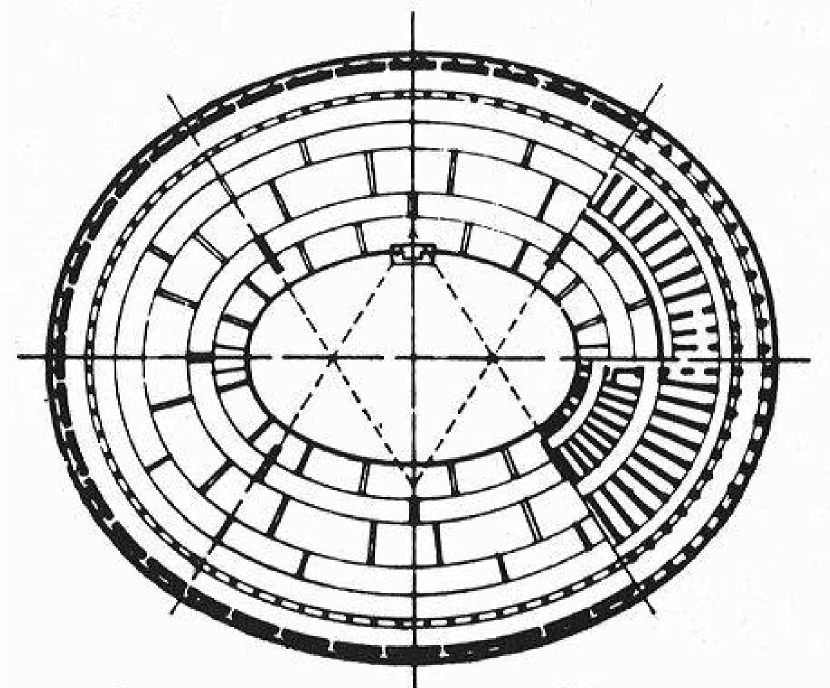
8/11/62 appears for the first time as the 4476th through 4480th digits.

Roman Coliseum

Elliptical arena axes dimensions

287 ft x 180 ft

$$\frac{287\text{ft}}{180\text{ft}} \approx 1.594 \approx 1.618 \approx \Phi$$



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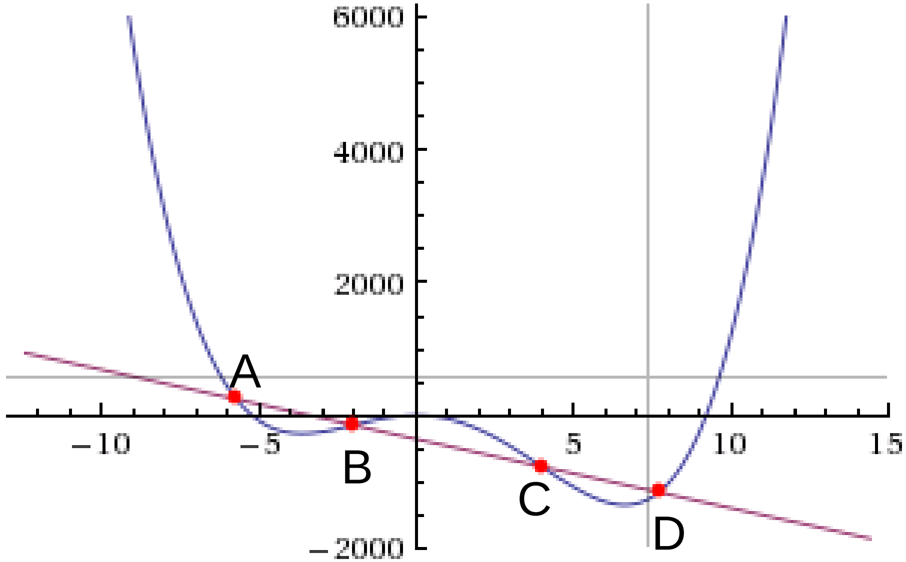


y = $x^4 - 4x^3 - 48x^2$; y = -104x – 352 ;
inflection points at -2 and 4.

$$A(1-3\sqrt{5}, 312\sqrt{5}-456)$$

$$B(-2,-144)$$

$$C(4,-768)$$

$$D(1+3\sqrt{5},-312\sqrt{5}-456)$$

$$AB = CD$$

$$\frac{BC}{AB} = \frac{AC}{BC} = \Phi$$

$$AB = CD \approx 385.6710365$$

$$BC \approx 624.0288455$$

$$AC \approx 1009.699877$$

Φ

Find a number that is one more than its reciprocal.

$$\text{Let } x = \text{the number}$$

$$x = \frac{1}{x} + 1$$

$$x^2 = 1 + x$$

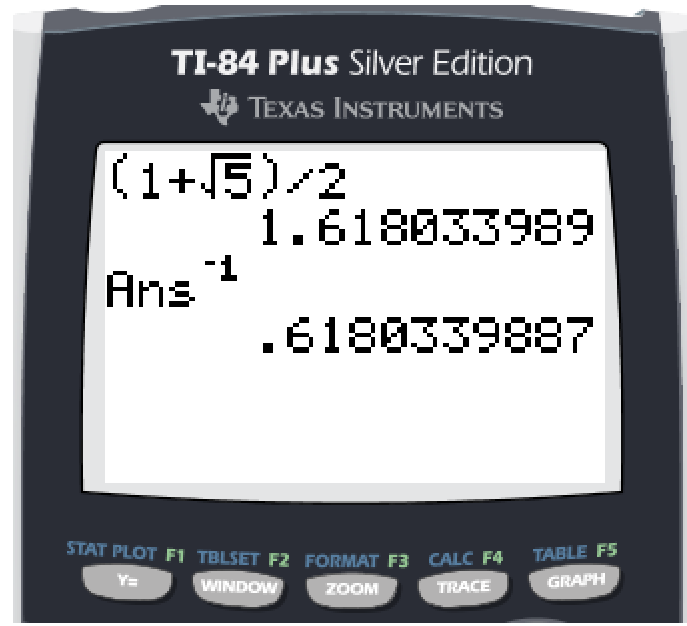
$$x^2 - x - 1 = 0$$

$$x = \frac{1+\sqrt{5}}{2} \approx 1.618033989...$$

$$x = \frac{1-\sqrt{5}}{2} \approx -0.618033989...$$

Φ

Find a number that is one more than its reciprocal.



$$\text{Let } x = \text{the number}$$

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$$x = \frac{1+\sqrt{5}}{2} \approx 1.618033989...$$

$$x = \frac{1-\sqrt{5}}{2} \approx -0.618033989...$$

Remember these ……

$$\Phi^2 = \Phi + 1$$

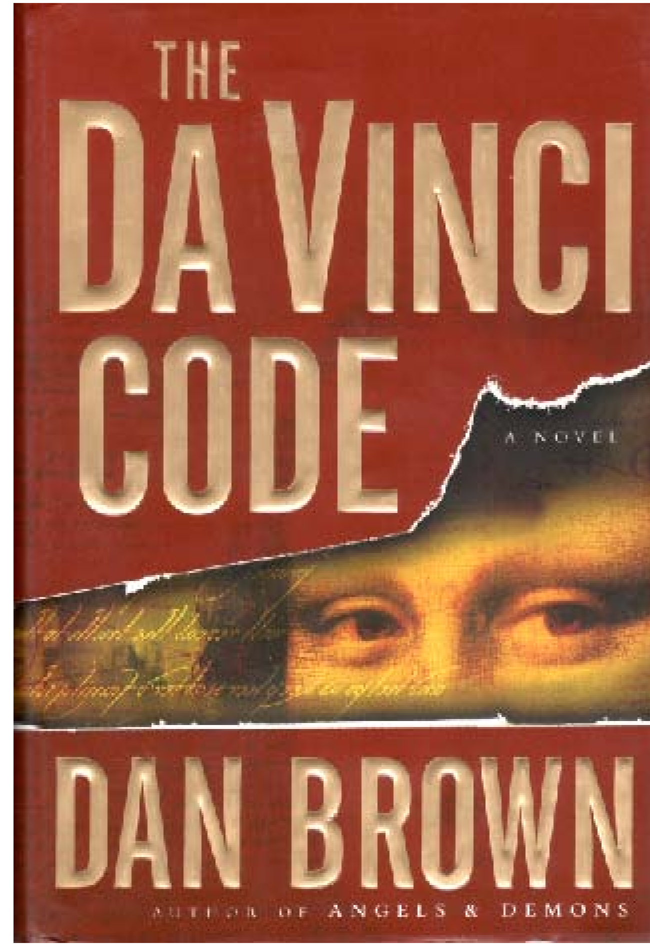
$$\Phi^2 - \Phi - 1 = 0$$

1.618

Numb3rs

http://www.youtube.com/watch?v=vFRTgr7MfWw

Φ

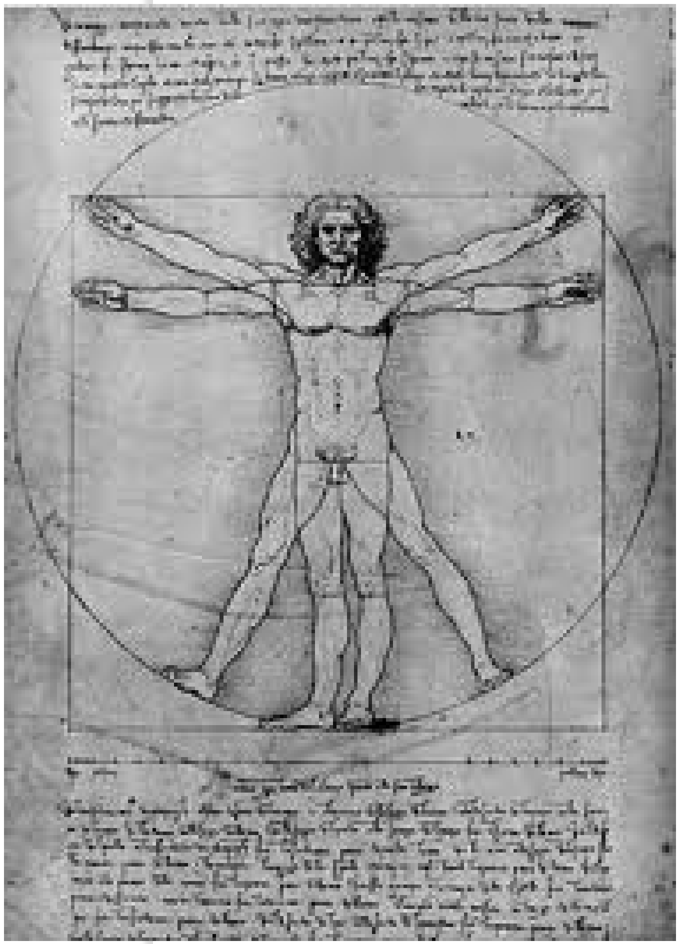


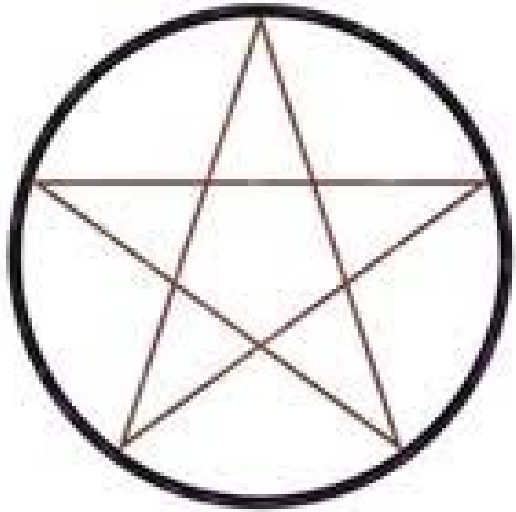
Audrey Tautou: Sophie (So Phee) Neveu
Tom Hanks: Robert Langdon

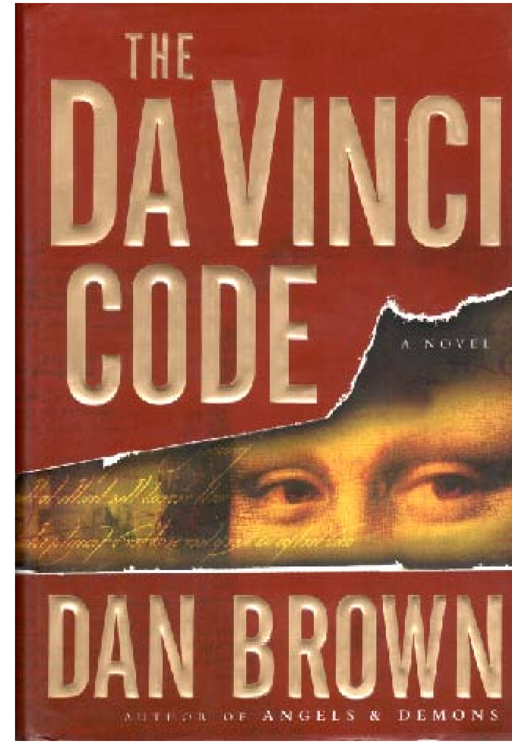
See pages 92-96 (Chapter 20).

Φ

Vitruvian Man

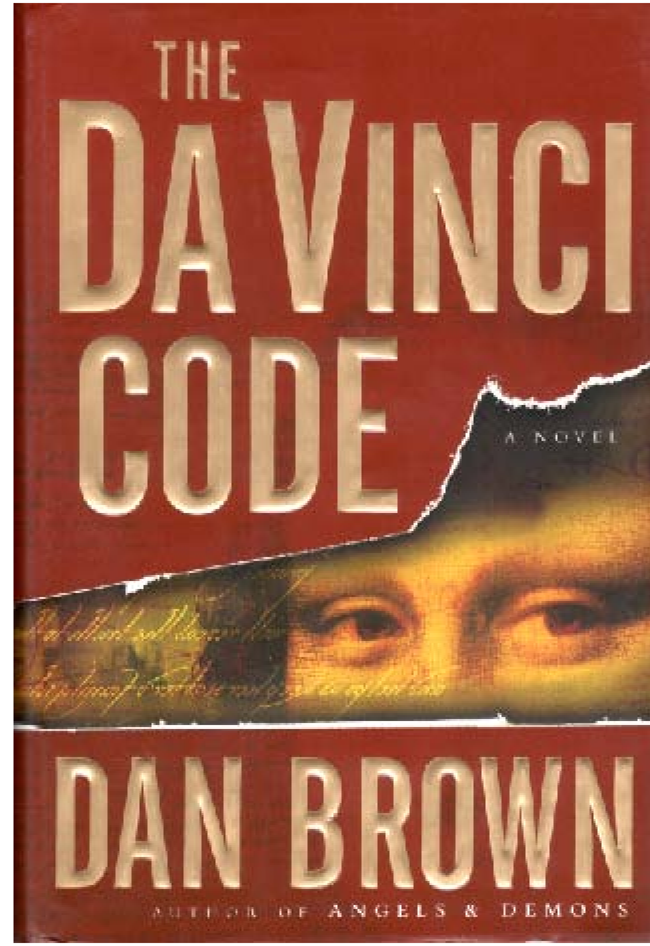






1, 1, 2, 3, 5, 8, 13, 21, …

Φ



Audrey Tautou: Sophie Neveu
Tom Hanks: Robert Langdon

The Divine Proportion: “PHI’s ubiquity in nature clearly exceeds coincidence, and so the ancients assumed the number PHI must have been preordained by the Creator of the Universe.”

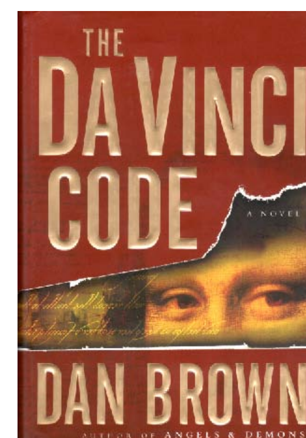
Φ

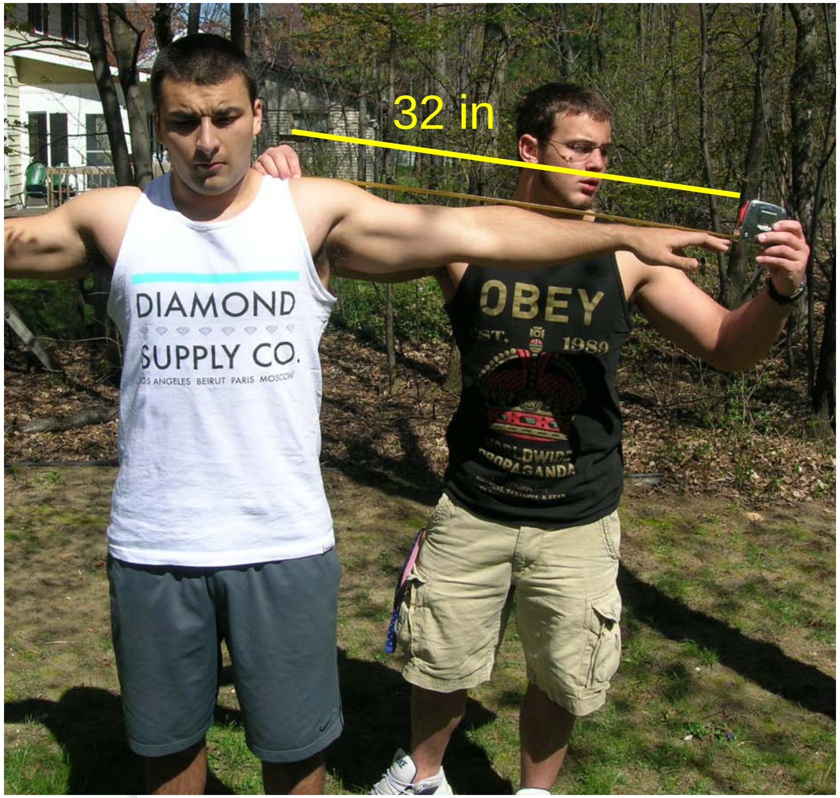
(Bellybutton height, Total height)

(Bellybutton to top of head, Bellybutton height)

(Tip of nose to top of head, Chin to top of head)

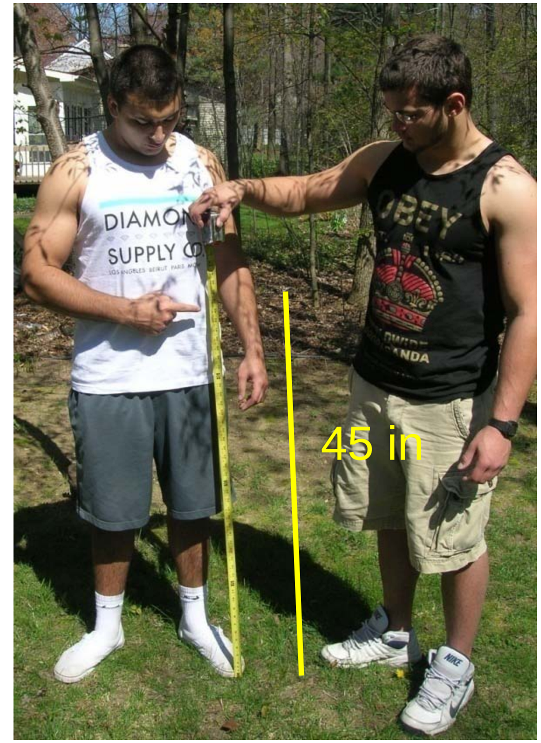
(Elbow to fingertips, Shoulder to fingertips)





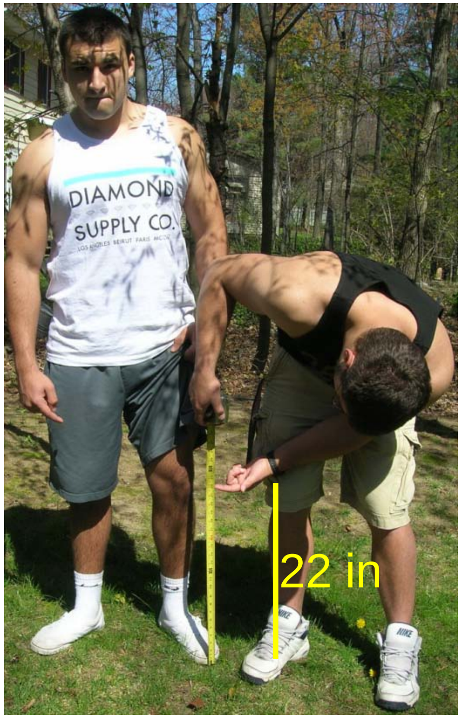
$$\frac{32\text{in}}{19\text{in}} \approx 1.684 \approx 1.618 \approx \Phi$$

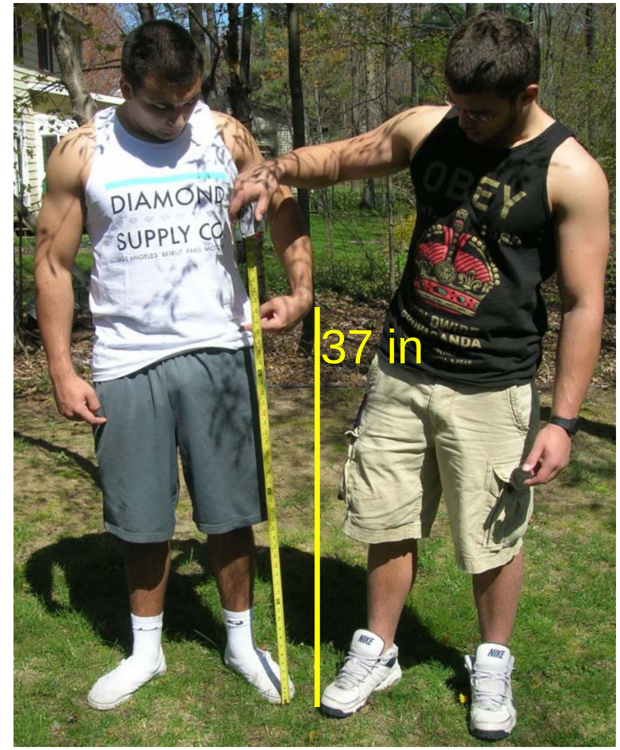
Φ



$$\frac{72\text{in}}{45\text{in}} \approx 1.6 \approx 1.618 \approx \Phi$$

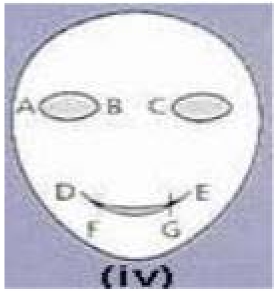
Φ

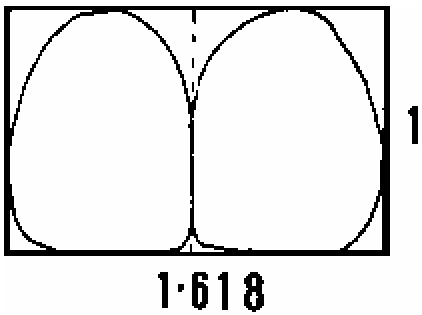




$$\frac{37\text{in}}{22\text{in}} \approx 1.682 \approx 1.618 \approx \Phi$$

Φ



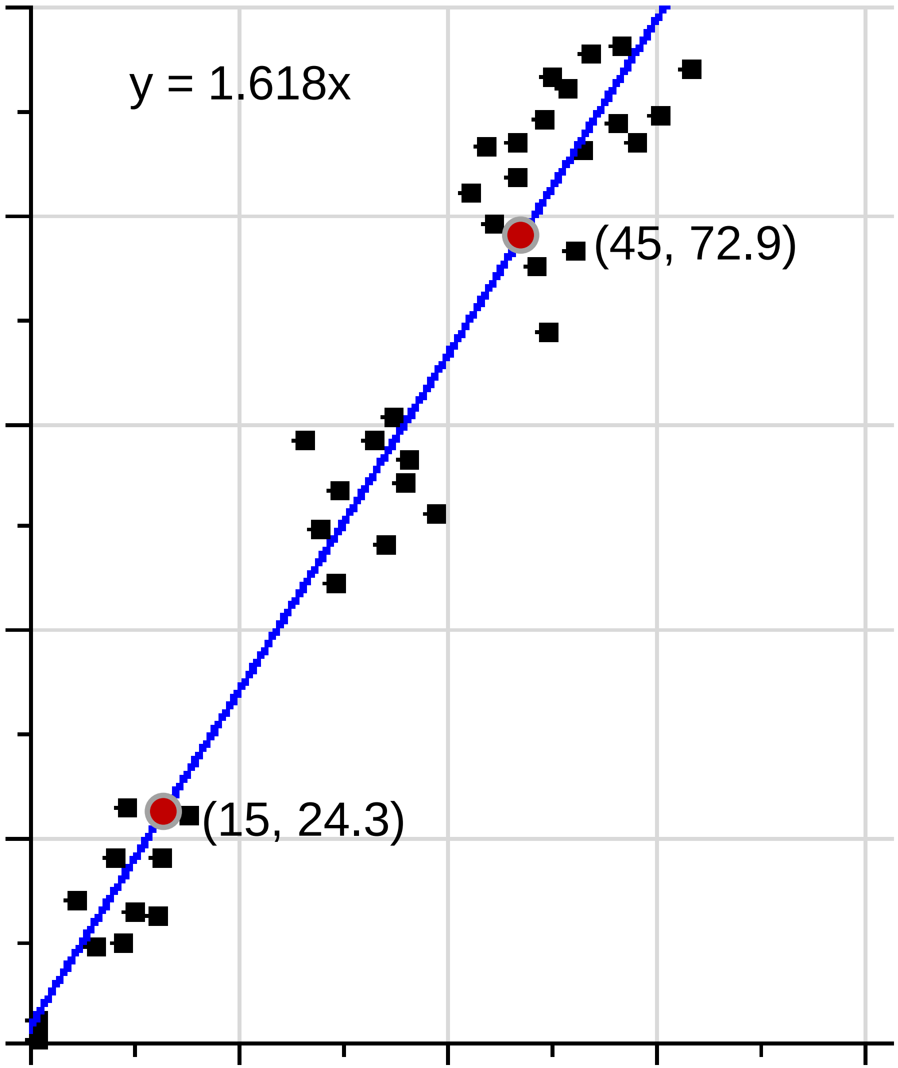


The ratio of the space between the eyes BC to the WHITE of the eyes AB is the golden ratio.

The ratio of the WHITE width FG of the Anterior Aesthetic Segment to the WHITE of the eyes AB is the golden ratio.

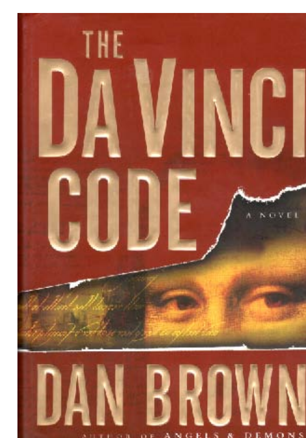
Φ

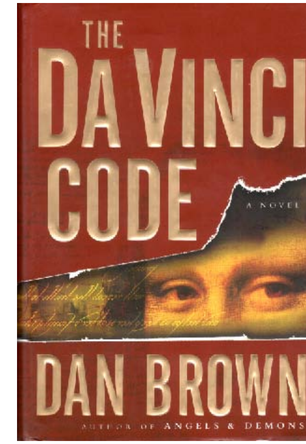
r = 0.971



Φ

Cephalopod Mollusk:
What is the ratio of each spiral’s diameter to the next?



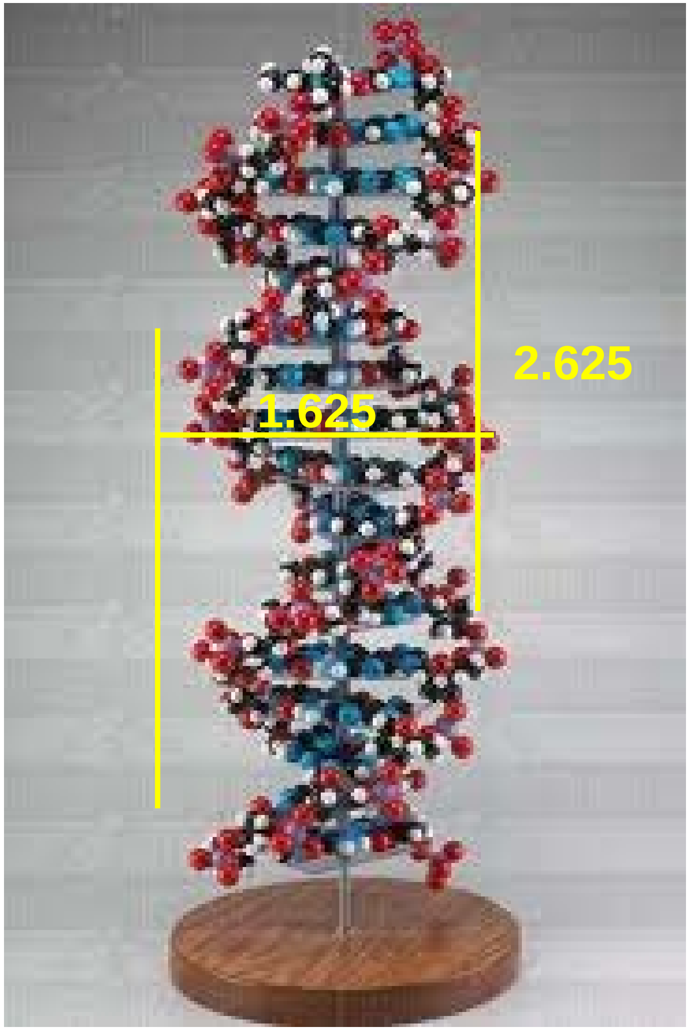


http://www.spur.org/blog/tag/bees

Approximately 95% of a bee hive will be workers, and the workers are female, so that leaves 5% males. If you divide 95 by 5 you get 19, which is nothing like 1.618. In severe winter, all the male bees die off, so then the ratio is infinite!

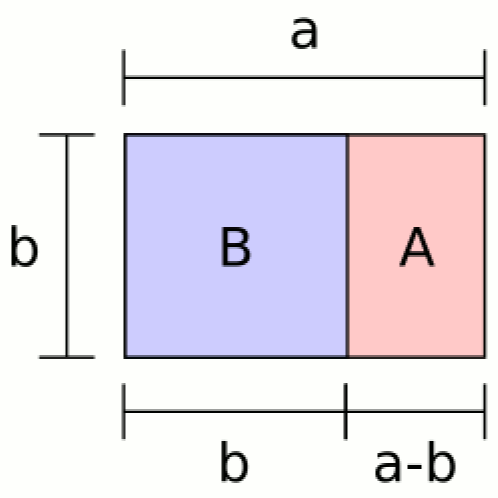
What does appear to be true, due to some peculiarities of the breeding processes of bees is that the numbers of male to female bees in the family history of a male bee at any given generation is a ratio of adjacent Fibonacci numbers.

DNA Double Helix



$$\frac{2.625}{1.625} \approx 1.615 \approx 1.618 \approx \Phi$$

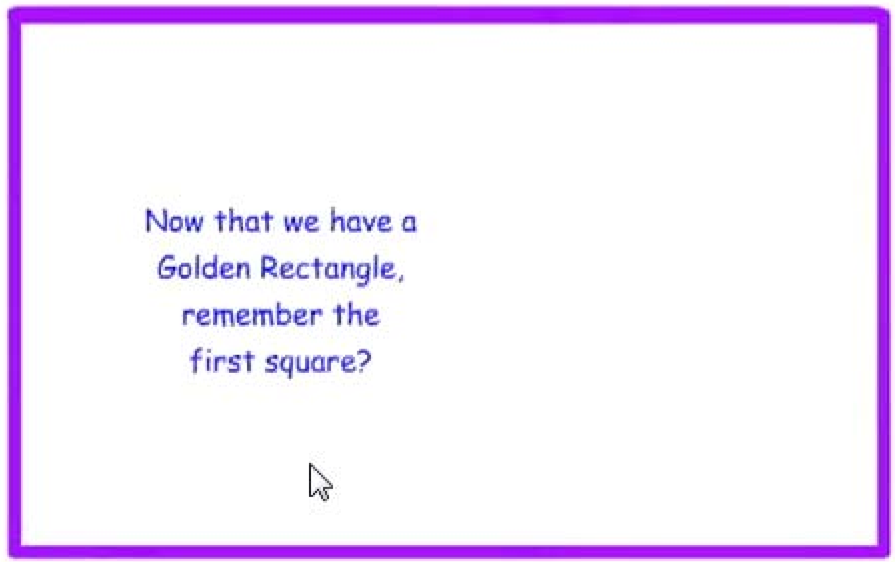
Φ



The Golden Rectangle

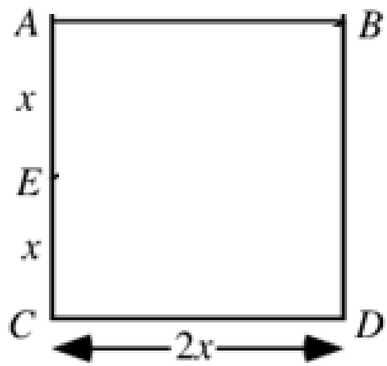
$$\frac{a}{b}=\frac{b}{a-b}=\Phi$$

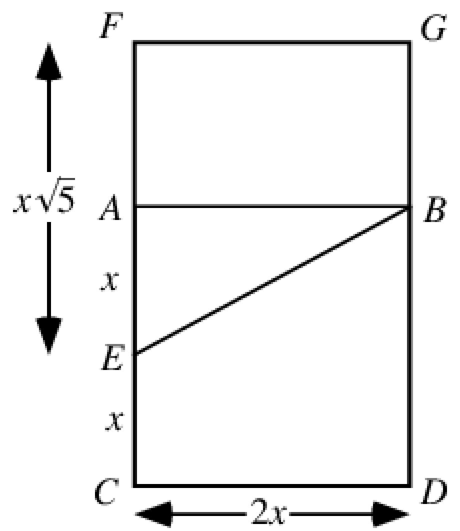
$$\frac{Area\,of\ B}{Area\,of\ A}=\frac{b^2}{b(a-b)}=\frac{b}{a-b}=\Phi$$



http://www.youtube.com/watch?v=TLxmLo0Zlg8

Φ





$$AB = 2x$$

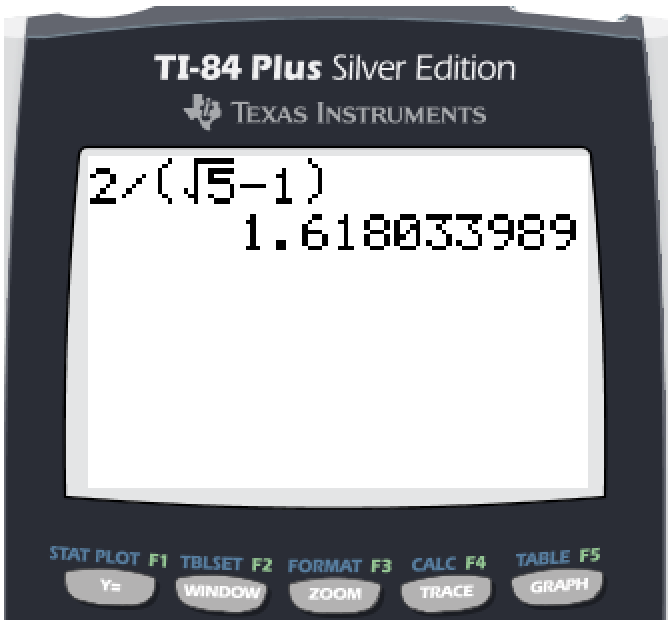
$$EB = x\sqrt{5}$$

$$AF = x(\sqrt{5} - 1)$$

$$CF = x(1 + \sqrt{5})$$

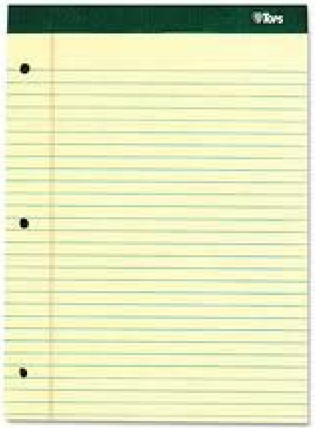
$$\frac{CF}{FG} = \frac{x(1+\sqrt{5})}{2x} = \frac{1+\sqrt{5}}{2} = \Phi$$

$$\frac{FG}{AF} = \frac{2x}{x(\sqrt{5}-1)} = \frac{2}{\sqrt{5}-1} = \Phi$$



Φ

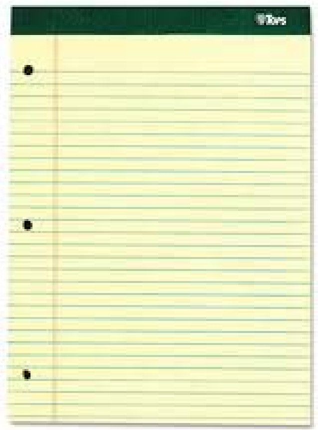
1 : 1.618



Letter

Legal

Which rectangle is more attractive?



Letter

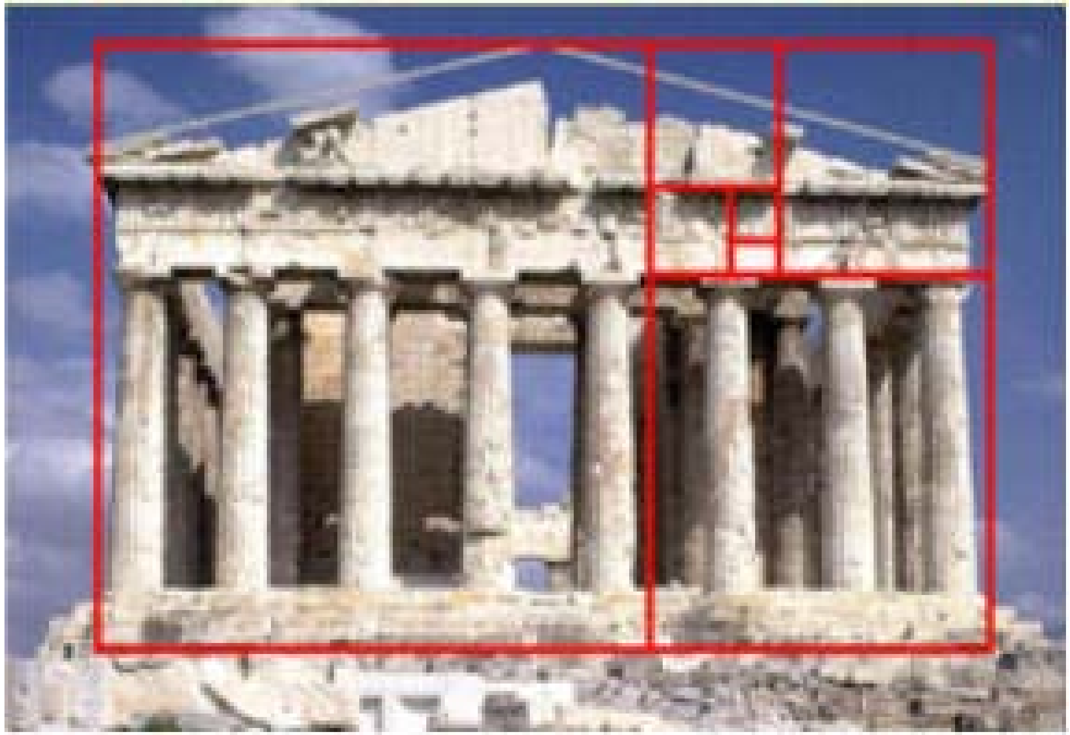
11/8.5 ≈ 1.29

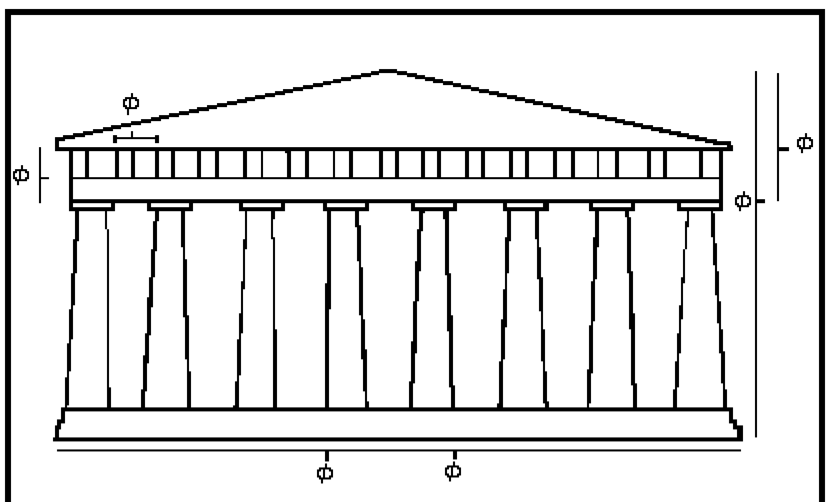
Legal

14/8.5 ≈ 1.65
More Golden!

Which logo is more attractive?

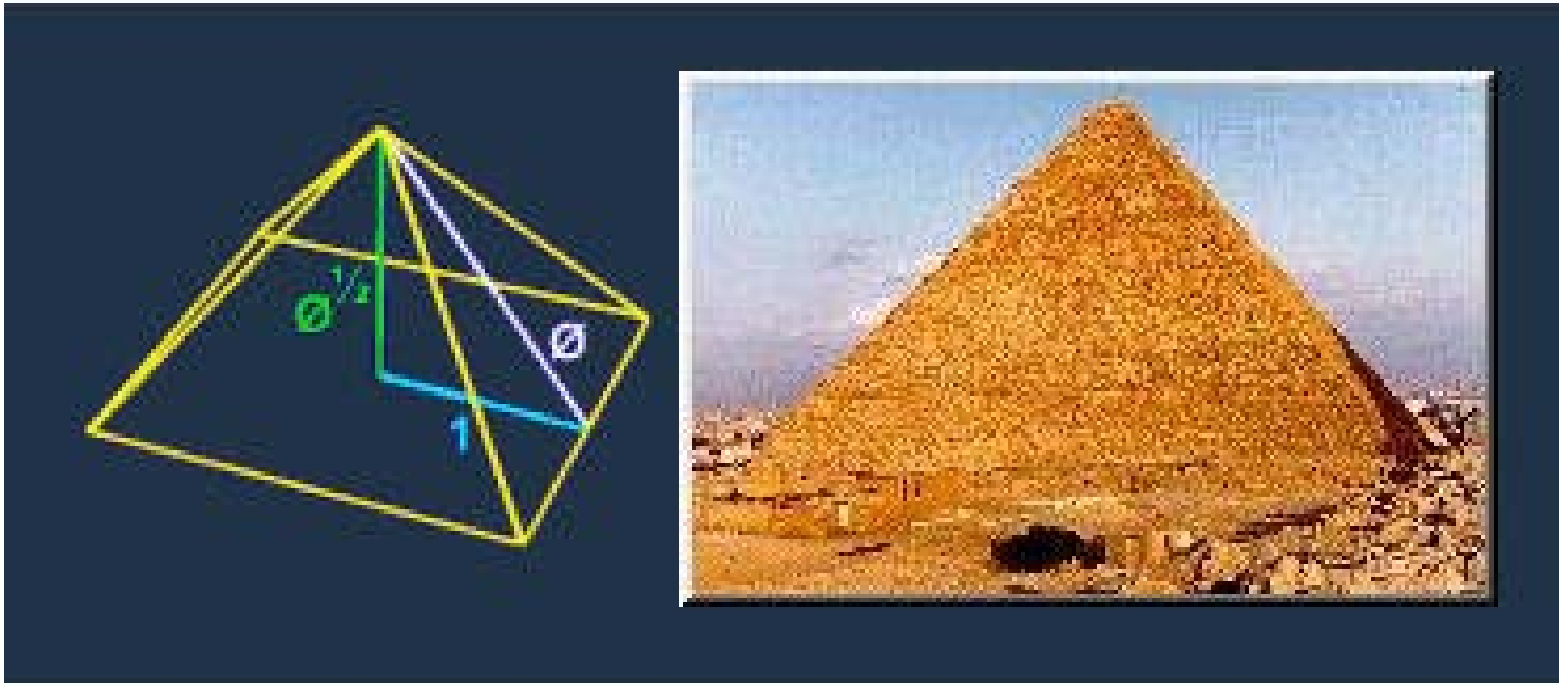
Φ





http://cuip.uchicago.edu/~dlnarain/golden/activity2.htm

Φ

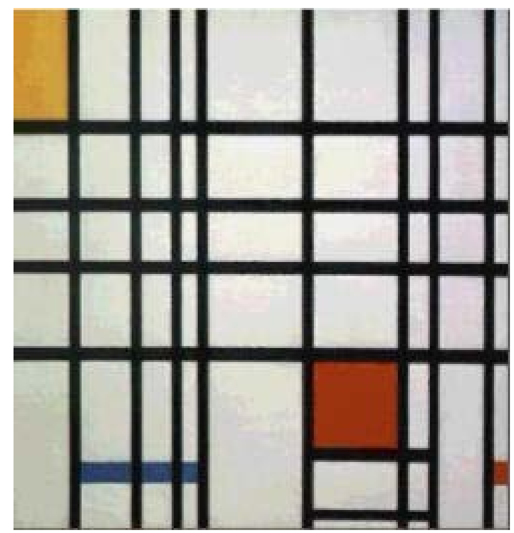


$$\left(\Phi^{1/2}\right)^2 + 1^2 = \Phi^2$$

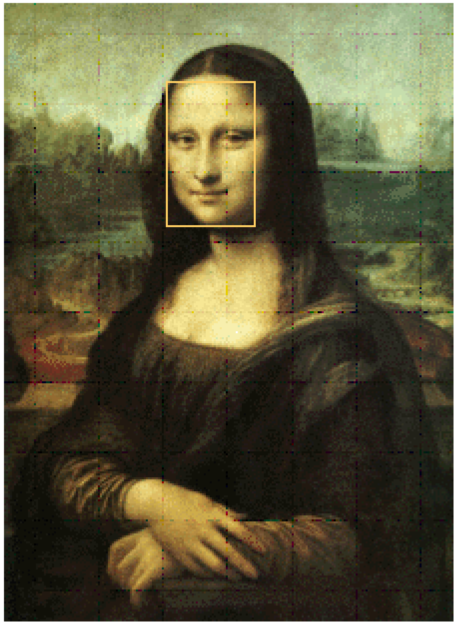
$$\Phi + 1 = \Phi^2$$

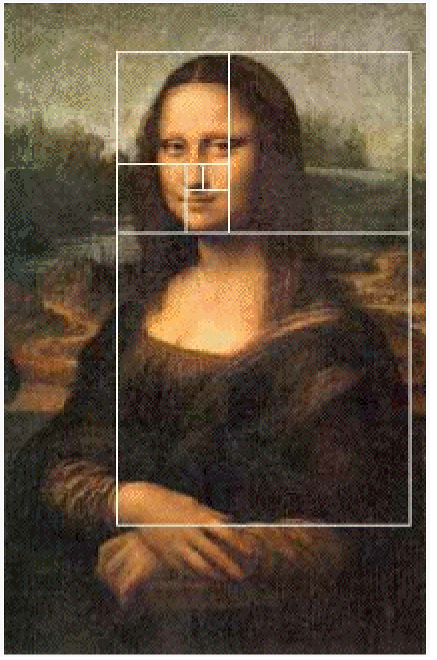
http://jwilson.coe.uga.edu/emat6680/parveen/ancient_egypt.htm

The Annunciation
Da Vinci
http://s3153554.blogspot.com/



Piet Mondrian





We can draw a rectangle whose base extends from the woman's right wrist to her left elbow and extend the rectangle vertically until it reaches the very top of her head. Then we will have a golden rectangle. Also, if we draw squares inside this Golden Rectangle, we will discover that the edges of these new squares come to all the important focal points of the woman: her chin, her eye, her nose, and the upturned corner of her mysterious mouth.

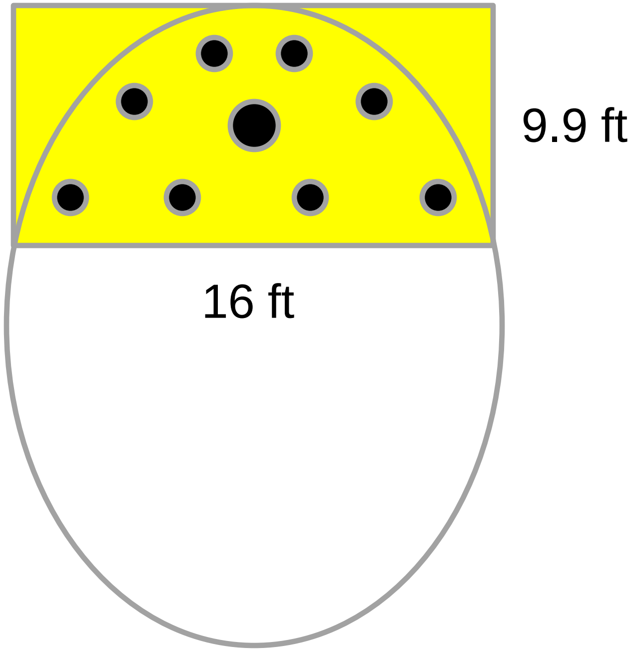
It is believed that Leonardo da Vinci, as a mathematician tried to incorporate of mathematics into art. This painting seems to be made purposefully line up with golden rectangle.

http://jwilson.coe.uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html

Landscaping: Elliptic region inscribe in a golden rectangle

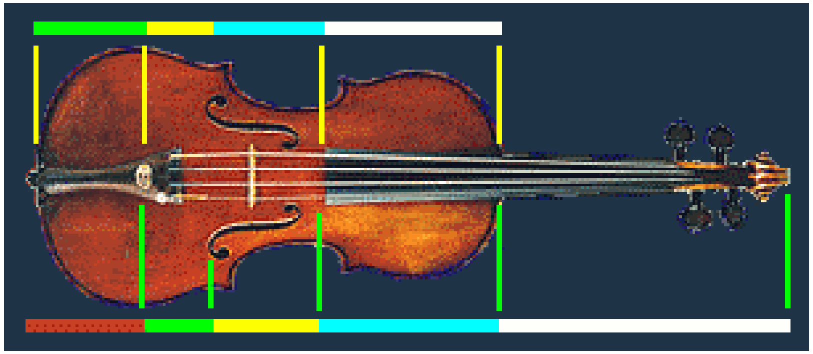
Width of the brick landscaping wall and length of the golden rectangle is 16 feet. The width of the golden rectangle is $16/\Phi \approx 9' \, 10.7''$. The foci of the ellipse fall on the axis of symmetry of the elliptic region on both sides of a 4-foot sidewalk.

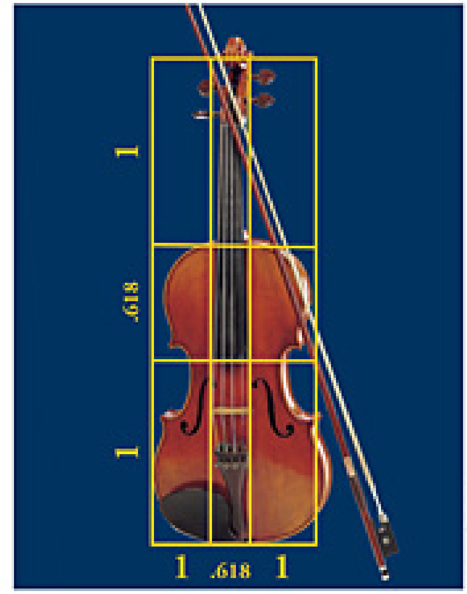
Landscaping: Elliptic region inscribe in a golden rectangle



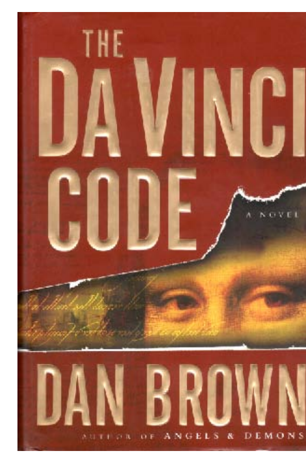
Φ

Stradivarius Violins



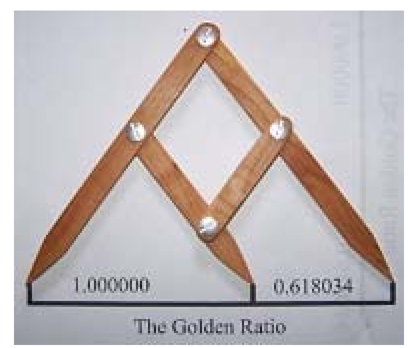


1 : .618



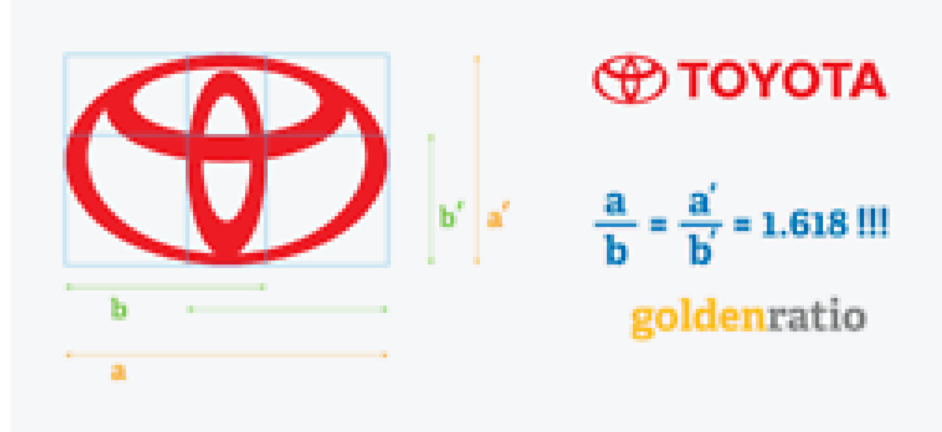
Φ

Object	Length	Width	Ratio
Index Card			
Photograph			
Picture Frame			
Textbook			
Door Frame			
Computer Screen			
TV Screen			
iPod			
iPad			
Credit Card			
Letter Paper			
Legal Paper			

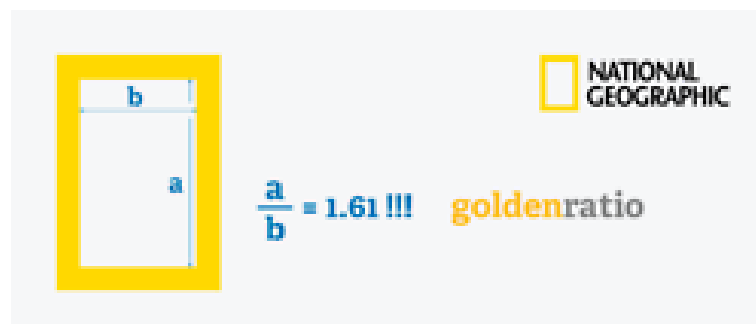


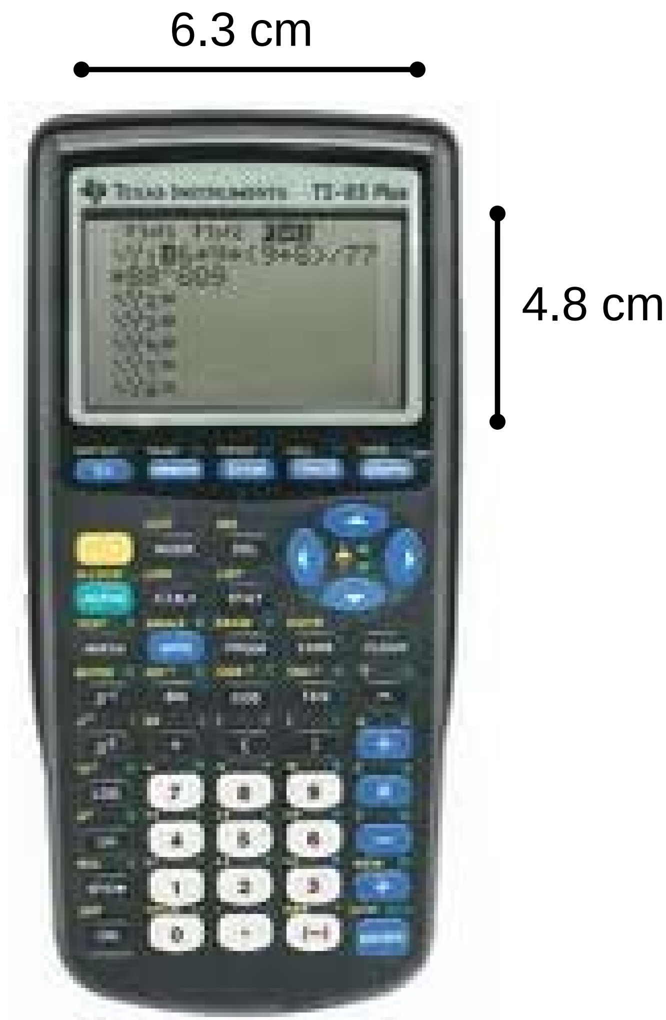
The Golden Ratio Gauge

Φ


TOYOTA
$\frac{a}{b} = \frac{a'}{b'} = 1.618$!!!
goldenratio

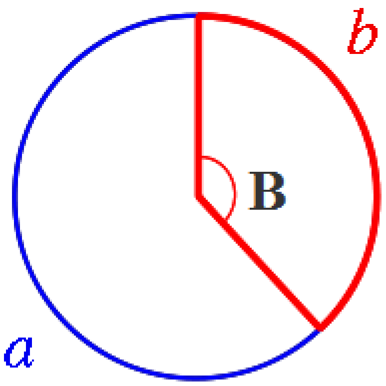

a
b
1
1.6
1.6
1
$\frac{a}{b} = 1.618$!!!
goldenratio


b
a
NATIONAL GEOGRAPHIC
$\frac{a}{b} = 1.61$!!!
goldenratio



$6.3/4.8 \approx 1.3125$

The Golden Angle



$$\frac{a+b}{a} = \frac{a}{b} = \phi$$

$$b(a+b) = a^2$$

$$\frac{b(a+b)}{b^2} = \frac{a^2}{b^2} = \phi^2$$

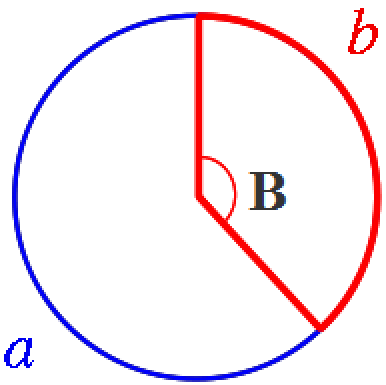
$$\frac{b}{a+b} = \frac{1}{\phi^2}$$

$$\frac{B}{b} = \frac{2\pi}{a+b}$$

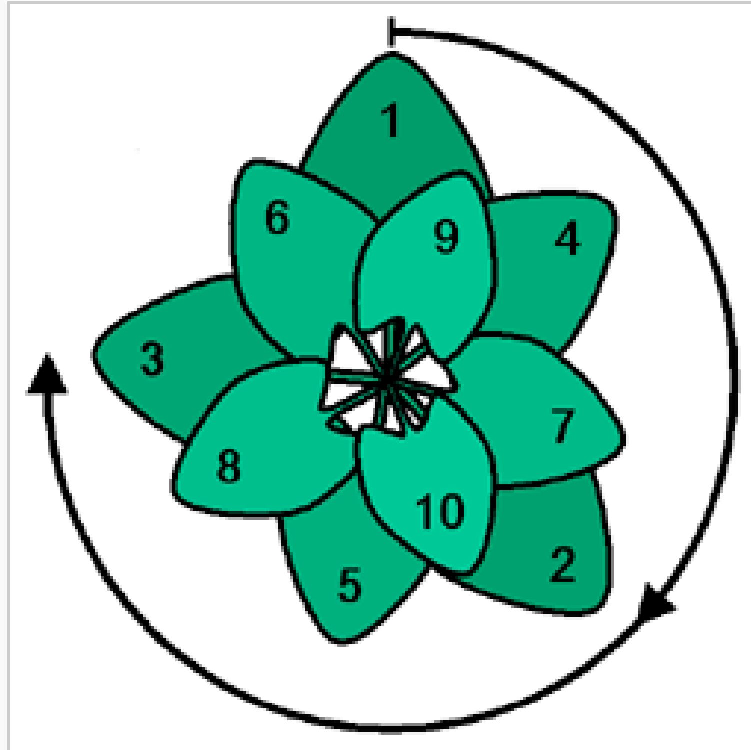
$$B = \frac{2\pi}{a+b} \bullet b = 2\pi \bullet \frac{b}{a+b} = 2\pi \bullet \frac{1}{\phi^2} = 2.39996323 \text{ radians} = 137.507764^\circ$$

$$\approx 2.4 \text{ radians} \approx 137.5^\circ$$

The Golden Angle



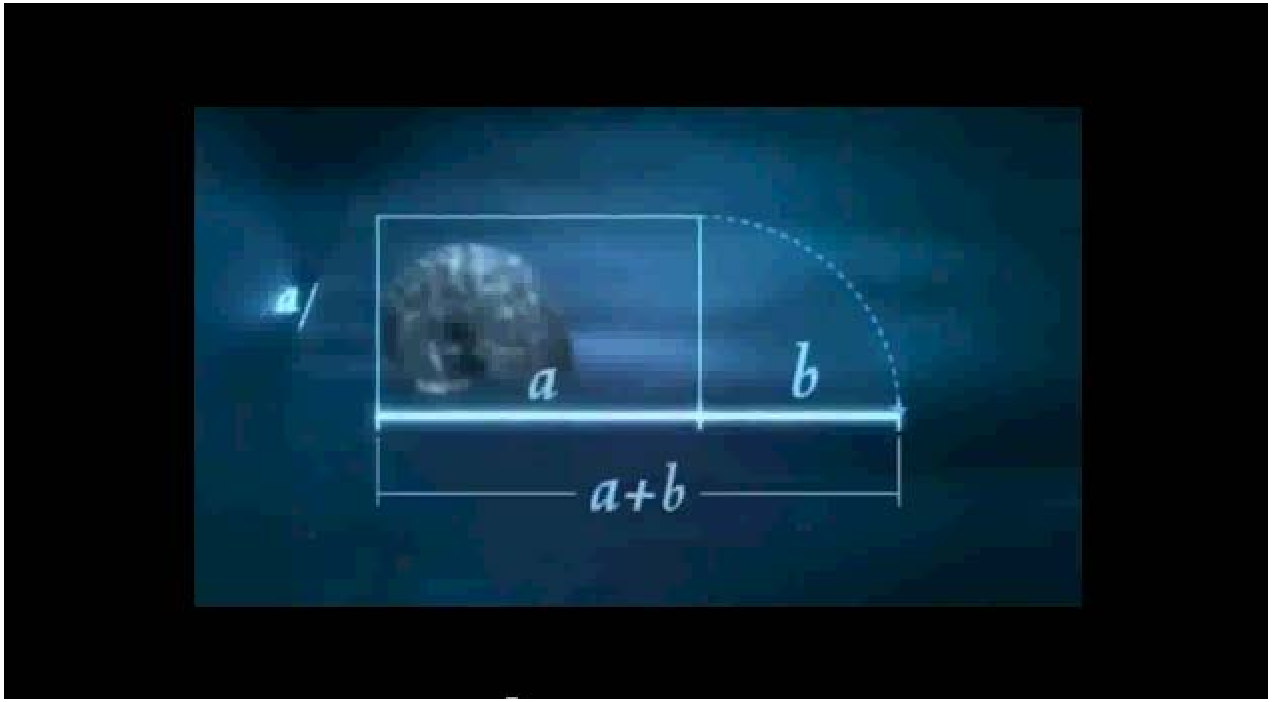
$$B \approx 137.5^\circ$$



The angle between successive florets in some flowers is the golden angle.

http://en.wikipedia.org/wiki/Golden_angle

Φ



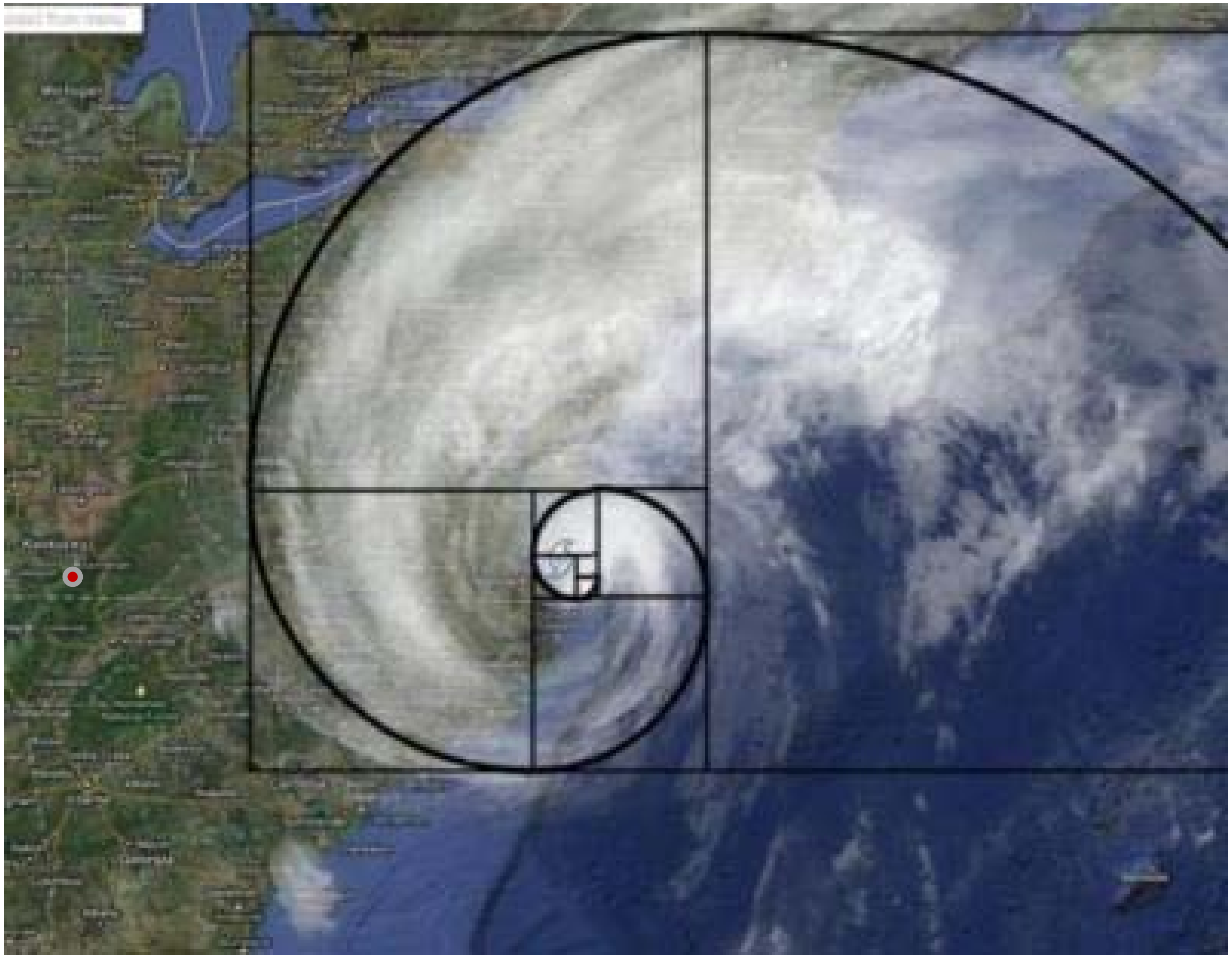
http://www.youtube.com/watch?v=eYDwWbDhCEg

$$\frac{55}{34} \approx 1.618 \approx \Phi$$

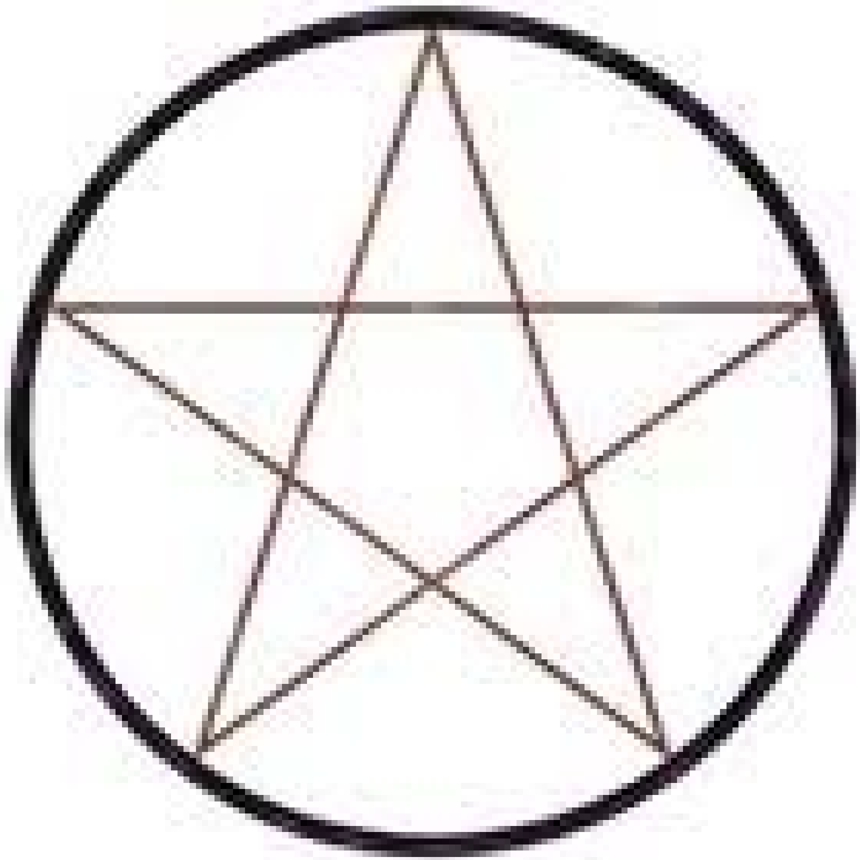
http://www.youtube.com/watch?v=SD-ZiqDvnKo

http://www.youtube.com/watch?v=sCeQ_5BPV20&feature=related

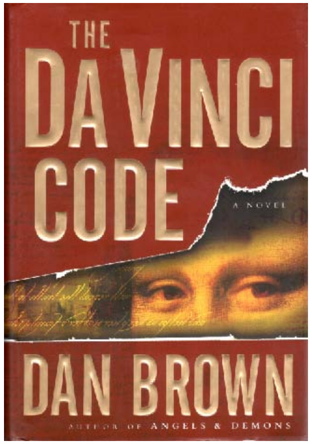
Φ



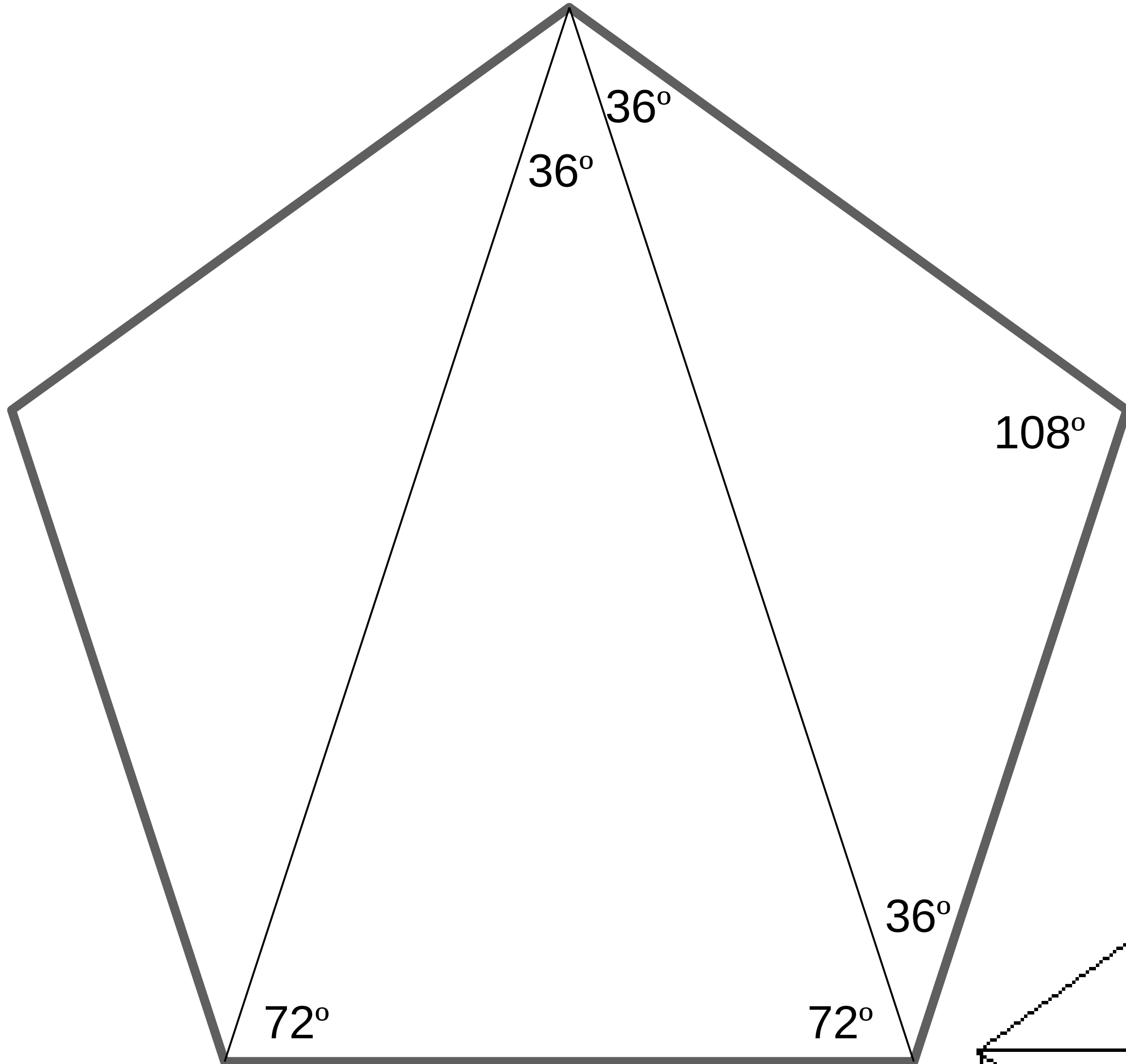
Pentagram or Pentacle

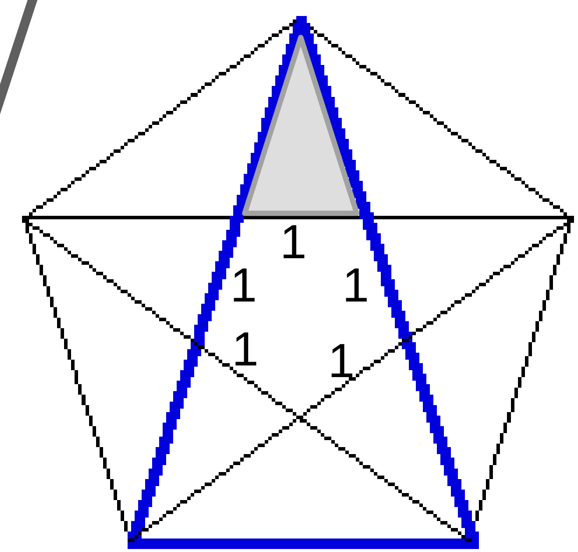


"… if you draw a pentagram, the lines automatically divide themselves into segments according to the Divine Proportion."

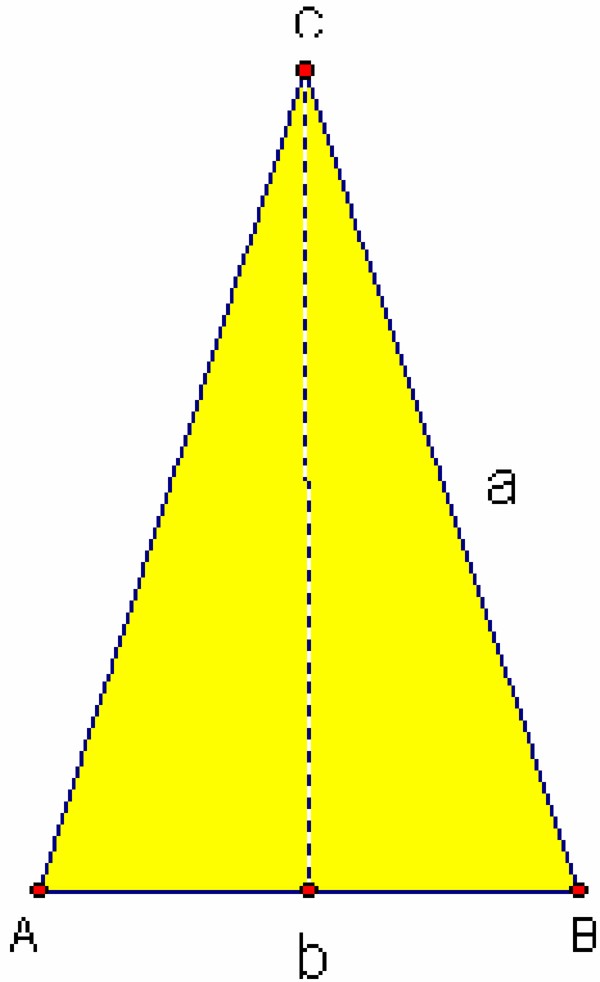


Φ



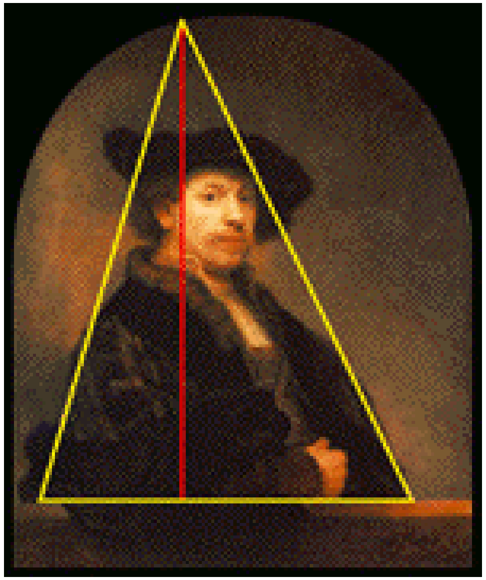


Φ



The golden triangle is an isosceles triangle such that the ratio of the hypotenuse *a* to base *b* is equal to the golden ratio,

$a / b = \Phi$

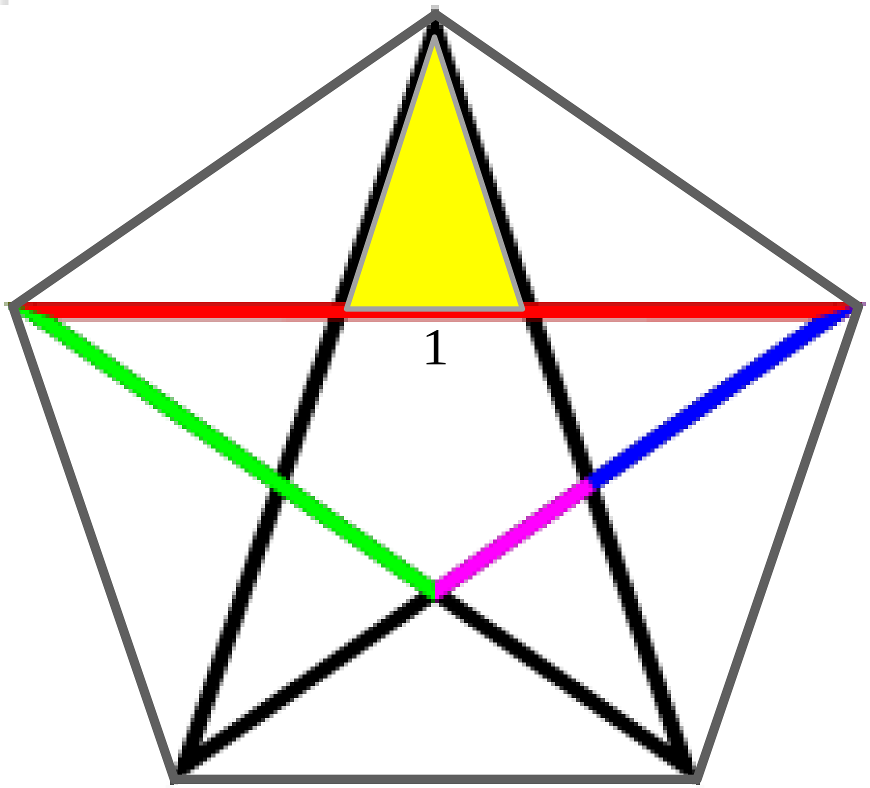


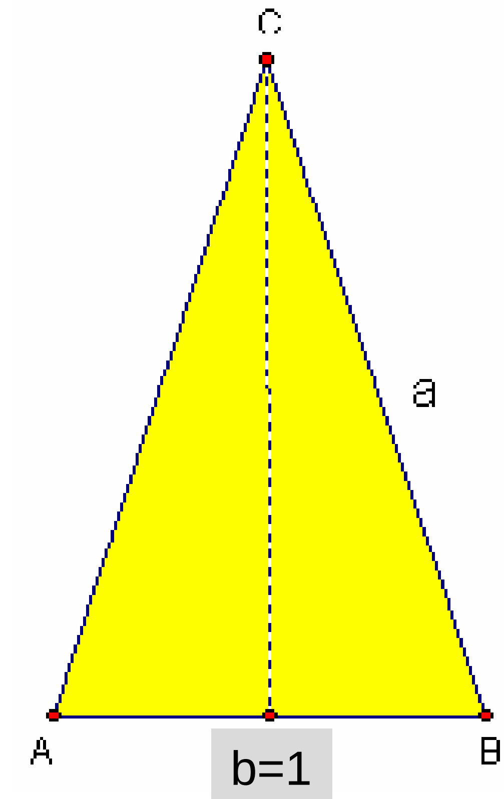
***self-portrait** by Rembrandt*

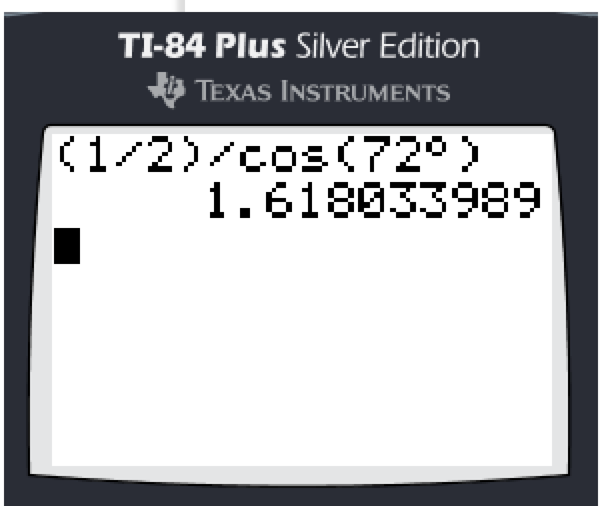
We can draw three straight lines into this figure. Then, the image of the feature is included into a triangle. Moreover, if a perpendicular line would be dropped from the apex of the triangle to the base, the line would cut the base in Golden Section.

http://jwilson.coe.uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html

Pentagram or Pentacle





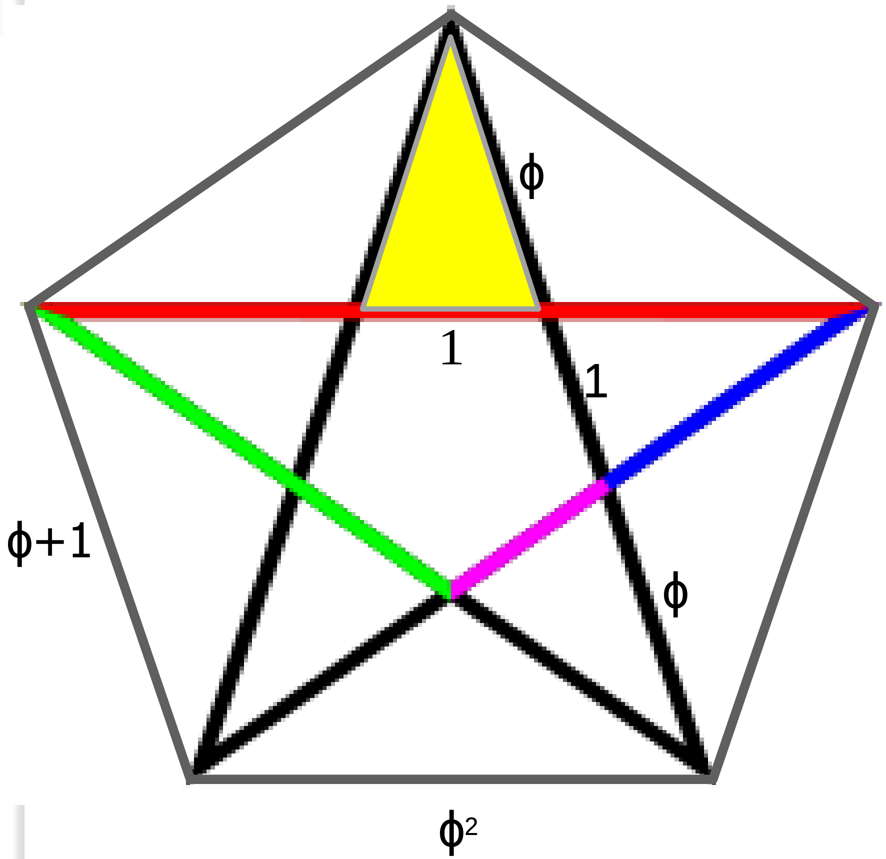


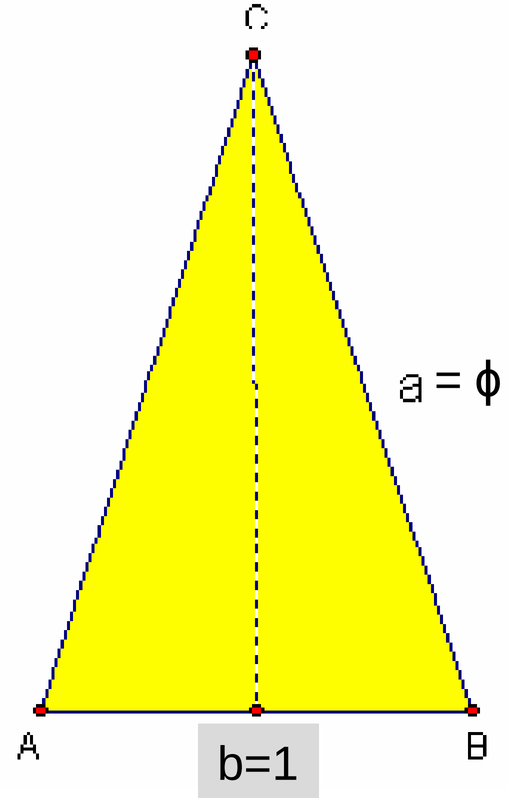
$$\angle CBA = 72^\circ$$

$$\cos(72^\circ) = \frac{1/2}{a}$$

$$a = \frac{1/2}{\cos(72^\circ)} = \Phi$$

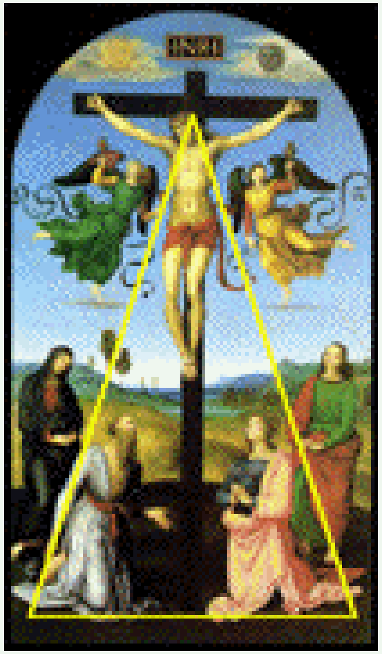
Pentagram or Pentacle

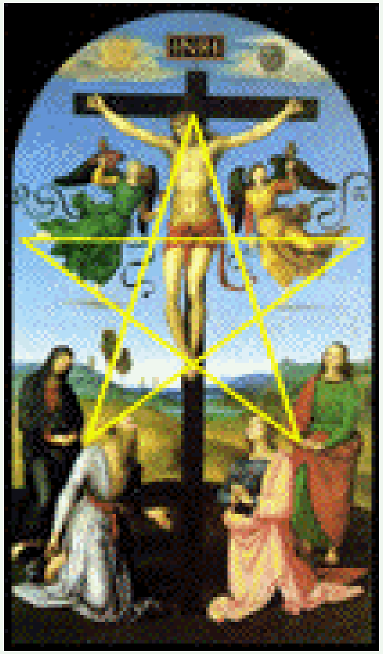




$$\phi^{2}+\phi = \phi(\phi+1) = \phi\ \phi^{2} = \phi^{3}$$

Φ



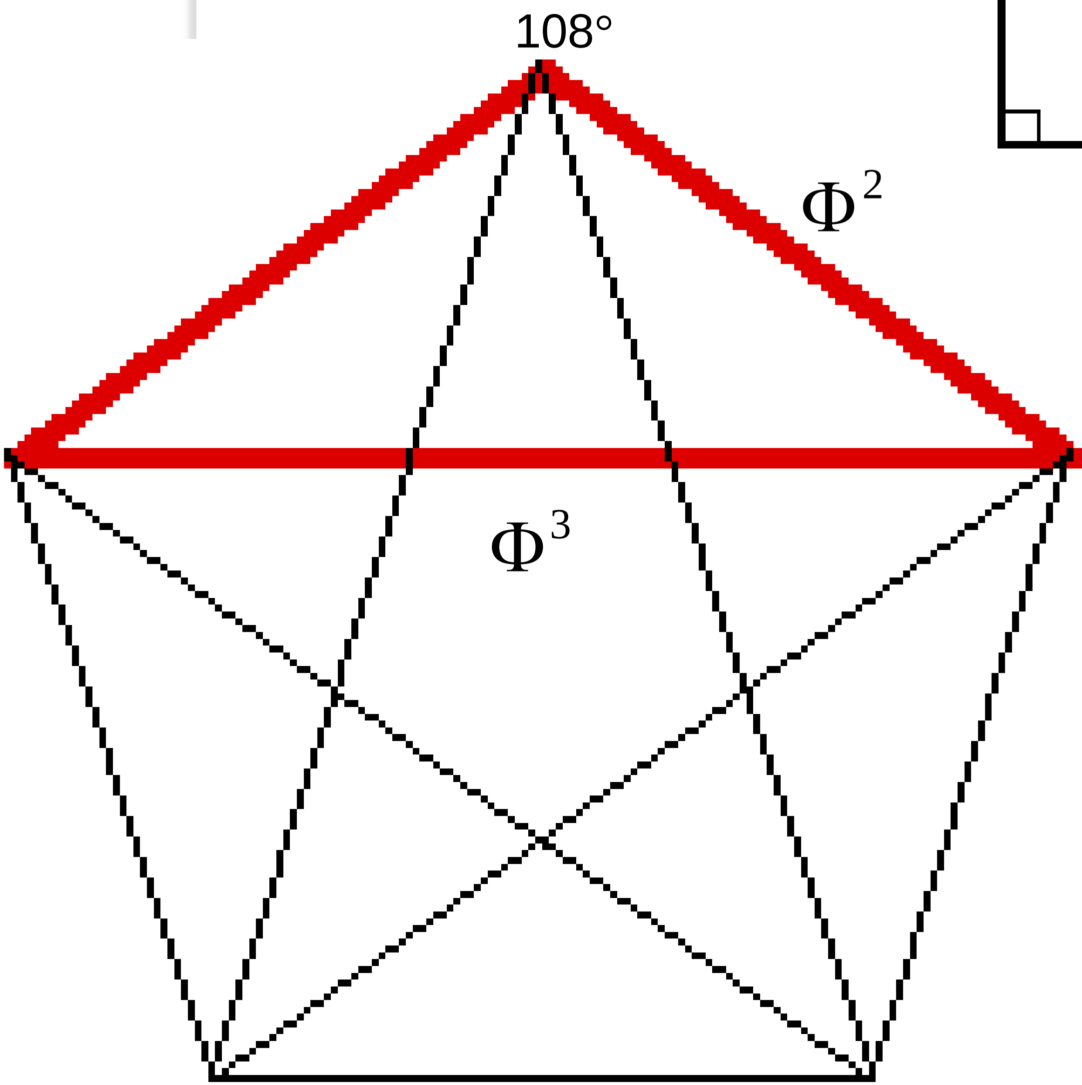


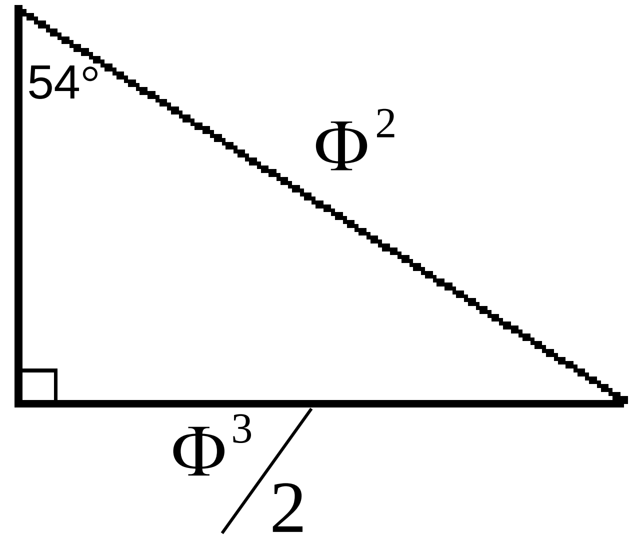
Crucifixion *by Raphael*

http://jwilson.coe.uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html

$\sin(54^{o}) =$

Φ

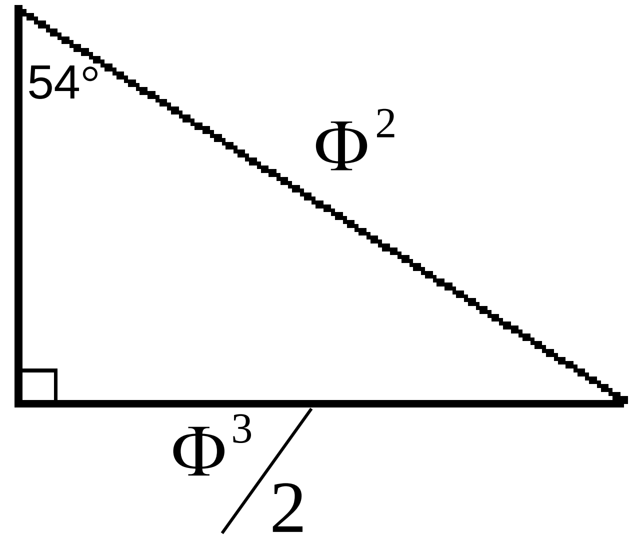


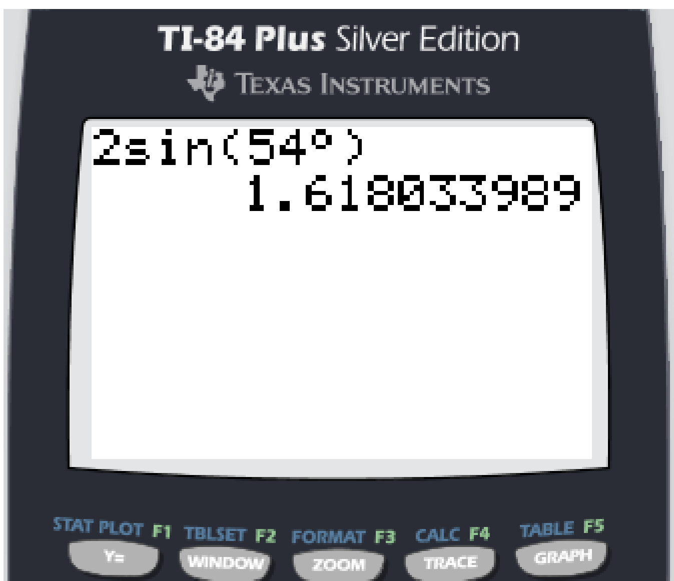


$$\sin(54^\circ) = \frac{\Phi^3/2}{\Phi^2} = \frac{\Phi}{2}$$

$$\Phi = 2\sin(54^\circ)$$

Φ





$$\sin(54^\circ) = \frac{\Phi^3/2}{\Phi^2} = \frac{\Phi}{2}$$

$$\Phi = 2\sin(54^\circ)$$

Φ

The Divine Proportion and The Number of the Beast

$\sin(666^{\circ}) =$

WolframAlpha

The Divine Proportion and The Number of the Beast

$$\sin(666^\circ)$$

$$= \sin(306^\circ)$$

$$= \sin(-54^\circ)$$

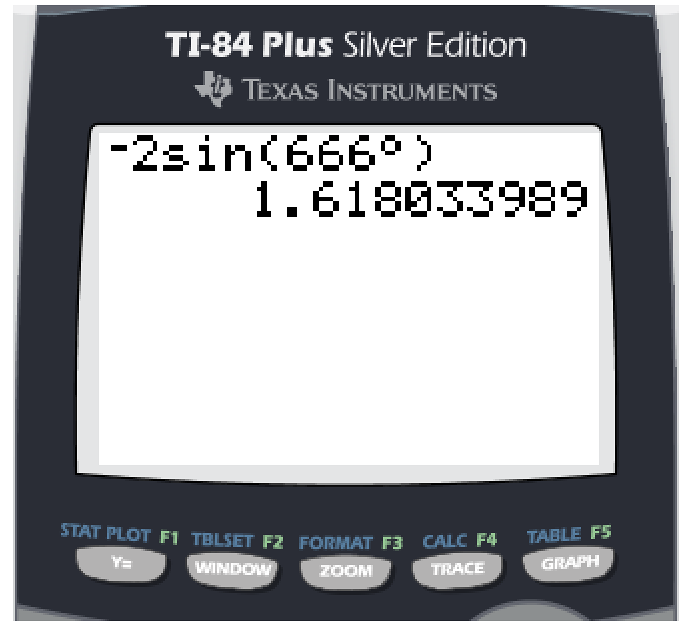
$$= -\sin(54^\circ)$$

$$= -\frac{\Phi}{2}$$

$$\Phi = -2\sin(666^\circ)$$

The Divine Proportion and The Number of the Beast

$$\sin(666^\circ)$$
$$= \sin(306^\circ)$$
$$= \sin(-54^\circ)$$
$$= -\sin(54^\circ)$$
$$= -\frac{\Phi}{2}$$
$$\Phi = -2\sin(666^\circ)$$



WolframAlpha

Φ

$$1 + \cfrac{1}{1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}$$

Φ

$$1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}$$

$$\text{Let } x = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ldots}}}}$$

Φ

$$1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}$$

$$\text{Let } x = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ldots}}}$$

$$\text{Let } x = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ldots}}}$$

Φ

$$\text{Let } x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}}}$$

x

$$x = 1 + \frac{1}{x}$$

$$x^2 = x + 1$$

$$x^2 - x - 1 = 0$$

$$x = \Phi$$

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}} = ??$$

$$x = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}}$$

$$x^2 = 1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}$$

Φ

$$x = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}}$$

$$x^2 = 1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}$$

$$x^2 = 1 + x$$

$$x = \Phi$$

$$\Phi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}}$$

Pascal’s Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

Φ

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

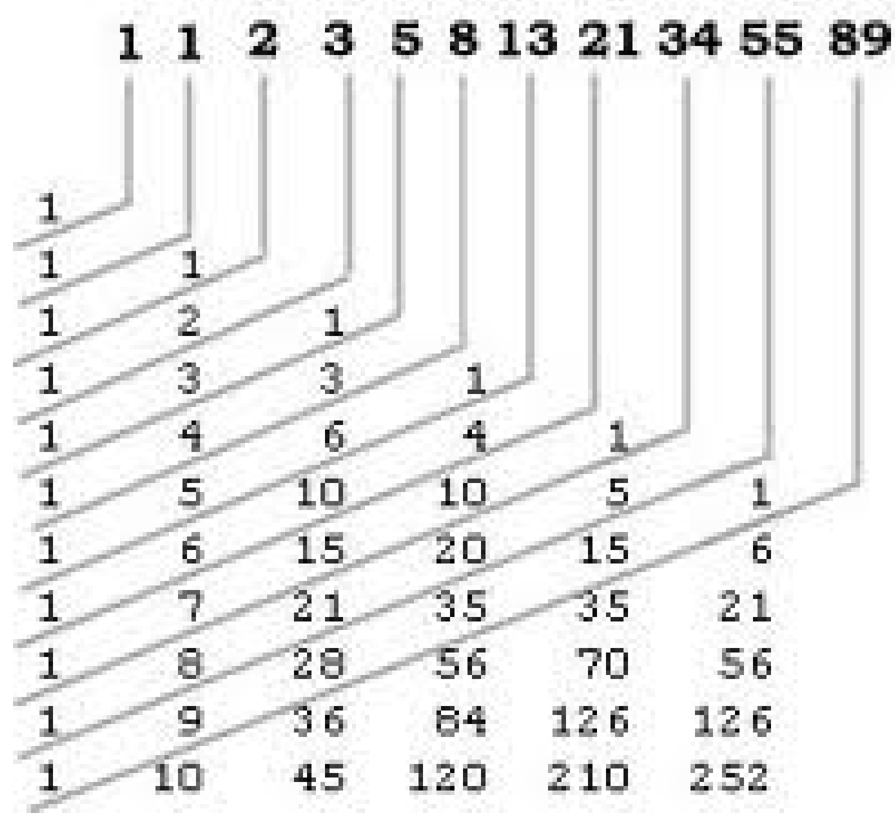
1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1



The Fibonacci Sequence

$$F_{n+2} = F_{n+1} + F_n$$
$$F_{n+1} = F_{n+1}$$

$$\text{If } u_k = \begin{bmatrix} F_{k+1} \\ F_k \end{bmatrix}$$

$$\text{Then } u_{k+1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} u_k$$

Φ

$$\text{Let } A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\text{Remember } x^2 = x + 1$$

$$\text{Try } A^2 = A + I$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A + I = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

… and the eigenvalues of $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \det\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right) = 0$$

$$\Rightarrow \det\left(\begin{bmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{bmatrix} \right) = 0$$

$$\Rightarrow -\lambda(1-\lambda) - 1 = 0$$

$$\Rightarrow \lambda^2 = \lambda + 1$$

$$\text{or } \lambda = \Phi$$

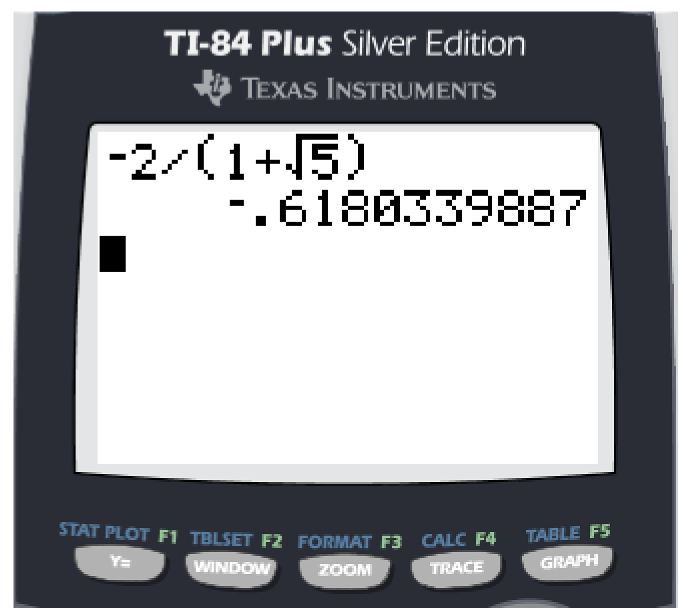
The Fibonacci Sequence closed form solution

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{-2}{1+\sqrt{5}}\right)^n\right) = \frac{1}{\sqrt{5}}\left(\Phi^n - \left(-\frac{1}{\Phi}\right)^n\right)$$

$$F_n \approx \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n\right)$$

X	Y_1	Y_2
1	1	.72361
2	1	1.1708
3	2	1.8944
4	3	3.0652
5	5	4.9597
6	8	8.0249
7	13	12.985

X	Y_1	Y_2
8	21	21.01
9	34	33.994
10	55	55.004
11	89	88.998
12	144	144
13	233	233
14	377	377



Φ

$$F_n \approx \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n\right)$$

$$\frac{F_{n+1}}{F_n} \approx \frac{\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1}\right)}{\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n}\right)} = \frac{1+\sqrt{5}}{2} = \Phi$$

Φ

List the first three terms of the sequence $\{F_n\}$.
Does the sequence F_n converge or diverge?

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{-2}{1+\sqrt{5}}\right)^n\right) = \frac{1}{\sqrt{5}}\left(\Phi^n - \left(-\frac{1}{\Phi}\right)^n\right)$$

Determine whether the series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{-2}{1+\sqrt{5}}\right)^n\right) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{5}}\left(\Phi^n - \left(-\frac{1}{\Phi}\right)^n\right)$$

Estimate the series below to 0.0001 using the alternating series remainder theorem:

$$\frac{13}{8}+\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(2n+1)!}{(n+2)!\,n!\,4^{2n+3}}$$

$$2\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{25}\right)^{k}\pi^{2k}}{(2k)!}$$

$$2\pi\sum_{k=0}^{\infty}\frac{(-1)^{k}\,3^{1+2k}\,10^{-1-2k}\,(-\pi)^{2k}}{(1+2k)!}$$

WolframAlpha

A number cubed is equal to the ratio of one more than the number to one less than the number.

$$Let\ \ x = the\, number$$

$$x^3 = \frac{x+1}{x-1}$$

$$x^3\left(x-1\right) = x+1$$

$$x^4 - x^3 = x+1$$

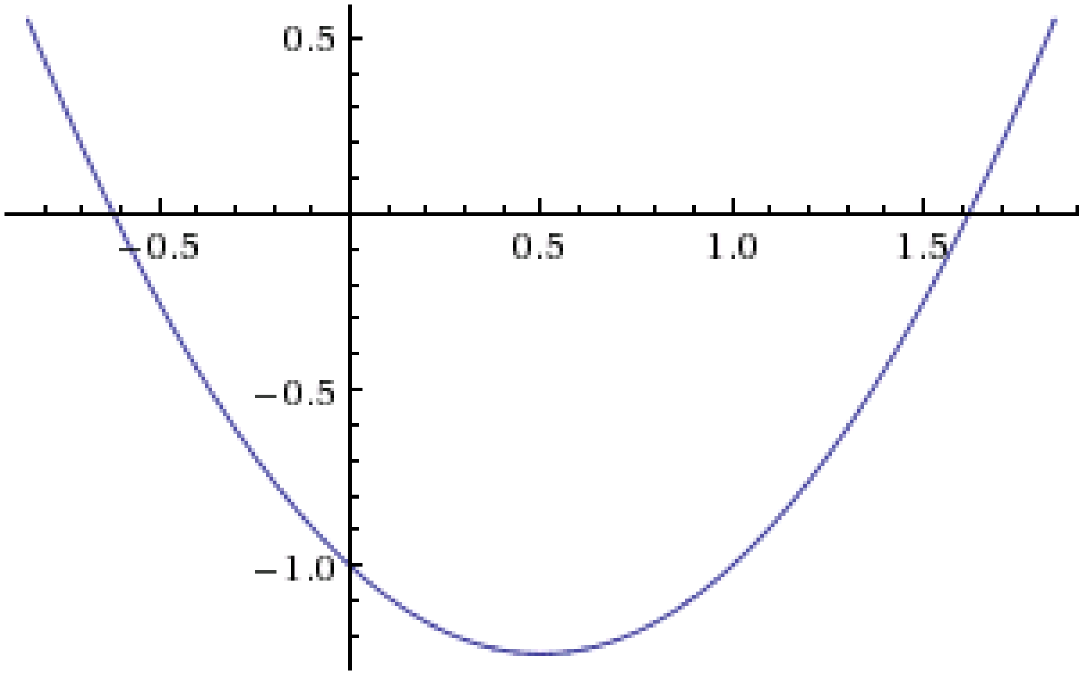
$$x^4 - x^3 - x - 1 = 0$$

$$\left(x^2 - x - 1\right)\left(x^2 + 1\right) = 0$$

$$x = \Phi$$

$$and\ldots$$

What is the largest x-intercept of the graph of $y = x^2 - x - 1$?

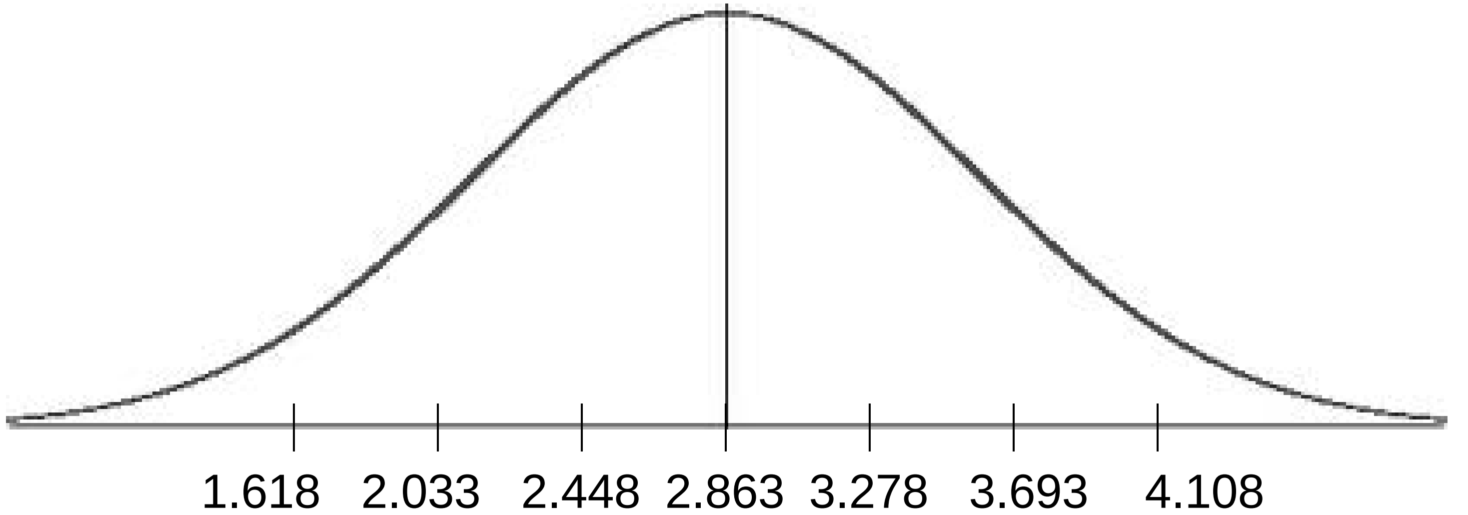


Find the extrema of f(x) = $2x^3 - 3x^2 - 6x + 9$.

$$f'(x) = 6x^2 - 6x - 6$$
$$= 6\left(x^2 - x - 1\right)$$

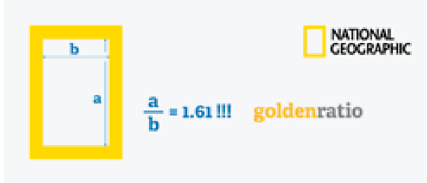
Find the inflection points of f(x) = $x^4 - 2x^3 - 6x^2 + 12x - 5$.

$$f'(x) = 4x^3 - 6x^2 - 12x + 12$$
$$f''(x) = 12x^2 - 12x - 12$$
$$= 12\left(x^2 - x - 1\right)$$

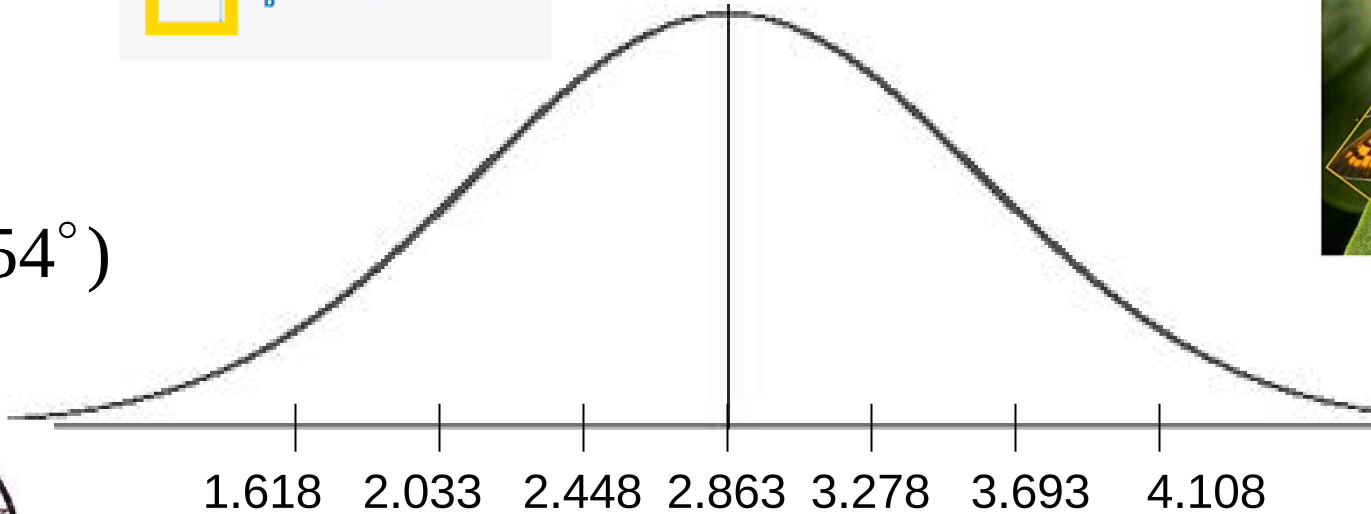


$\mu = 2.863$
$\sigma = 0.415$

Φ



$$2\sin(54^\circ)$$

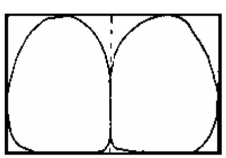


1 : 1.618

$$\Phi=\frac{1+\sqrt{5}}{2}$$

$$-2\sin(666^\circ)$$

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}}=??$$



1

1·618

$$1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}$$

Φ

$$e^{i\pi} + 1 = 0$$

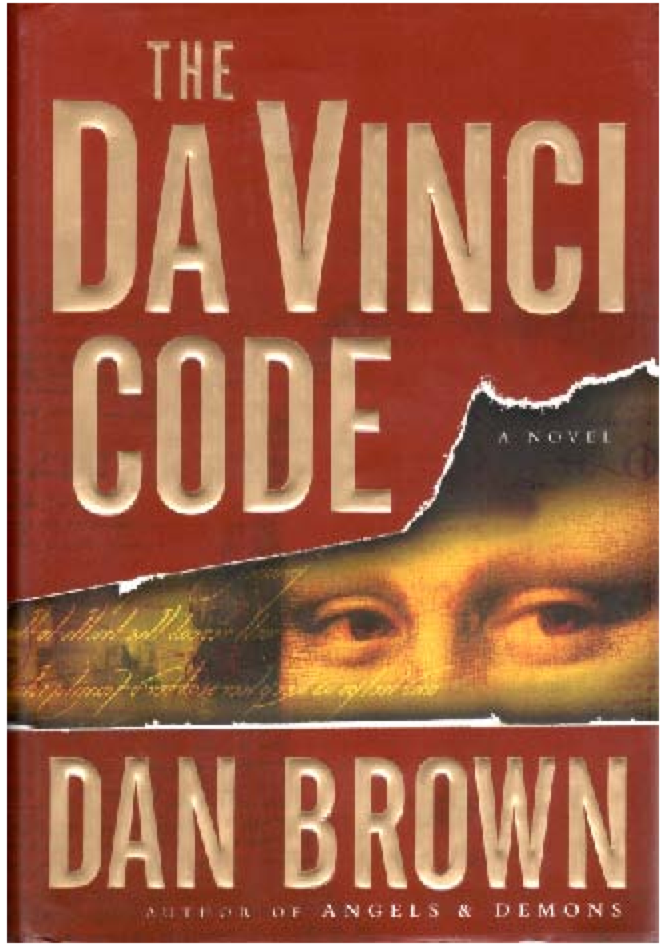
(Euler's Identity)

$$e^{i\pi} + \Phi^0 = \ln(1)$$

$$e^{i\pi} + \Phi^0 = 0$$

$$e^{i\pi} + \Phi - \frac{1}{\Phi} = 0$$

Φ



“PHI is one H of a lot cooler than PI”

Φ

Thank You!

Jim Ham

Delta College

http://www.jimham.net/goldenratio.pdf

jaham@delta.edu

Kentucky Mathematical Association of Two-Year Colleges

AMATYC

40th Annual Conference
November 8-11, 2014
Nashville, TN